Final exam - loppukoe

Introduction to generalized quantifiers

16.12.2004

1. Show that the dual of a monotone bijection closed generalized quantifier is always monotone and bijection closed.
2. One of the players has a (rather easy) winning strategy in

   $$\mathrm{EF}_9^{\geq\frac{1}{2}}((\{0, ..., 1024\}, <, 512), (\{0, ..., 1024\}, <, 513))?$$

   Describe this winning strategy.
3. What are the orbits of the structure $(\mathbb{N}, R)$, where $R \subseteq \mathbb{N} \times \mathbb{N}$ is the relation $R = \{(n, m) \in \mathbb{N} \times \mathbb{N} : n < 10\}$? Show that the orbits are really orbits and that there are no other orbits.
4. Write a sentence of $L_{\omega\omega}(\exists^{\geq\omega})$ of quantifier rank 3 which holds in a graph if and only if at least two vertices have infinitely many neighbors.
5. Show that there is no sentence of $L_{\omega\omega}(\exists^{\geq\omega})$ of quantifier rank 2 which holds in a graph if and only if at least two vertices have infinitely many neighbors.

Final exam - loppukoe

Introduction to generalized quantifiers

20.1.2005

1. Show that if $A \sim B \cup C$, then there is $D \subseteq A$ such that $B \sim D$.
2. One of the players has a winning strategy in

$$\mathrm{EF}_n^{\exists^{\geq\omega}}((\mathbb{Z},<),(\mathbb{R},<))?$$

   Describe this winning strategy.
3. What are the invariant subsets of the structure $(\mathbb{N}, R, 0, 1, 2)$, where $R \subseteq \mathbb{N}$ is the set of odd numbers.
4. Let $L = \{R\}$, where R is binary. Write a sentence of $L_{\omega\omega}(\exists^{\geq\frac{1}{2}})$ which holds in a finite L-structure if and only if more than 50% of the elements are in relation R with at least 50% of the elements.
5. Let $L = \{R\}$, where R is binary. Show that there is no sentence of $L_{\omega\omega}(\exists^{\geq\frac{1}{3}})$ which holds in a finite L-structure if and only if more than 50% of the elements are in relation R with at least 50% of the elements.