Final exam - loppukoe

Independence friendly logic II

22.5.2006

1. Write the Σ^1_1-sentence

$$\exists R(\exists x_0 Rx_0 \wedge \forall x_0(\neg Rx_0 \vee \exists x_1(Rx_1 \wedge Sx_0x_1)))$$

   in Skolem Normal Form, and give a sentence of DF which is logically equivalent to it.

2. Suppose ϕ is a Σ^1_1-formula. Show that there is a Σ^1_1-formula ψ such that for all $\mathcal{M}$ and s:

$$\mathcal{M} \models_s \psi \text{ if and only if } \mathcal{M} \models_s \forall x_n \phi.$$

3. Let L be the empty vocabulary. Suppose ϕ is an L-sentence of DF such that ϕ is true in exactly the infinite L-structures. Show that there is a natural number n such that the sentence ϕ is non-determined in all finite models of size $\geq n$.

4. Give two models $\mathcal{M}$ and $\mathcal{N}$ such that every sentence of DF true in $\mathcal{M}$ is true in $\mathcal{N}$ but some sentence of DF true in $\mathcal{N}$ is not true in $\mathcal{M}$.

5. Show that there cannot be a recursive function $f : \mathbb{N} \to \mathbb{N}$ such that for all ϕ in DF the following conditions are true:

   (a) $f(\ulcorner\phi\urcorner) = \ulcorner\psi\urcorner$ for some first order sentence ψ

   (b) ϕ is valid if and only if $f(\ulcorner\phi\urcorner) = \ulcorner\psi\urcorner$ for some first order sentence such that ψ is valid.

Independence friendly logic II
14.6.2006

1. Write the sentence

$$\exists x_0 R x_0 x_1 \vee \forall x_0 S x_0 x_2$$

in Skolem Normal Form, and prove the equivalence.

2. Show in team logic that a team X is of type

$$(\phi \otimes \top) \oplus \bot$$

if and only if for every $Y \subseteq X$ there is $Z \subseteq Y$ such that Z is of type ϕ.

3. Let $\mathcal{N} = (\mathbb{N}, +, \times, 0, 1)$ and let L be the vocabulary of $\mathcal{N}$. Show that there is no first order $\tau(x_0)$ in vocabulary L such that for all first order L-sentences ϕ we have

$$\mathcal{N} \models \phi \text{ if and only if } \mathcal{N} \models \tau(\ulcorner \phi \urcorner).$$

You can use the Gödel Fixed Point Theorem.

4. Suppose player **II** has a winning strategy in the Ehrenfeucht-Fraïssé game of length n for DF for the models $\mathcal{M}$ and $\mathcal{N}$ in a relational vocabulary L. Show that if ϕ is a sentence of DF of qr-rank at most n in the vocabulary L and $\mathcal{M} \models \phi$, then $\mathcal{N} \models \phi$.

5. Show that if ϕ is a sentence in DF, then the validity of ϕ can be expressed by a Π_2-sentence in the Levy-hierarchy.