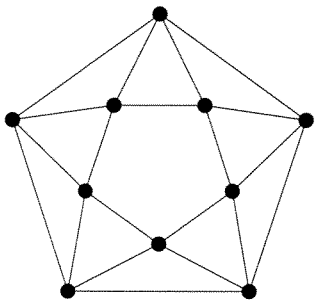
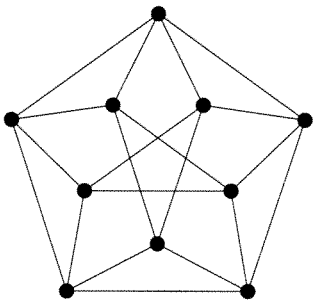
Department of Mathematics and Statistics
Graphs
Final Exam
17.12.2007

1. Are the following two graphs isomorphic? Justify your answer!





2. Consider the graph with 600 vertices shown on the reverse side of the paper.
   (a) Determine the number of edges of the graph. The mere answer is not enough, show also the calculation!
   (b) Does the graph have a strongly connected orientation? Justify your answer!

3. We define a digraph $S = (\mathcal{V}, \mathcal{E})$ by setting $\mathcal{V} = [12] (= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\})$ and $\mathcal{E} = \{(x, y) : x, y \in [12], x + y \neq 11$ and either $y - x = 1$ or $x - y = 2\}$.
   Justify your answer to the following questions:
   (a) Does S have a root (i.e., a vertex from which there is a walk to any other vertex)?
   (b) Is S strongly connected?
   (c) Is there a Hamiltonian path in S?
   [Hint: To draw S, divide the vertices into two groups according to parity.]

4. We define a graph $G = (\mathcal{V}, \mathcal{E})$ by setting

$$\mathcal{V} = \mathcal{P}[5] \ (= \{A : A \subset \{1, 2, 3, 4, 5\}\}) \ \text{ and } \ \mathcal{E} = \{\{A, B\} : A, B \in \mathcal{P}[5] \text{ and } A \cap B = \emptyset\}\,.$$

   Is there an Eulerian trail or circuit in G? Justify your answer!