

Department of Mathematics and Statistics  
Geometry  
2. midterm exam  
18.5.2005

- (1) Determine those ellipses whose focus is  $(0, 0)$  and directrix is  $x = 2$ . Does the point  $(1, 0)$  belong to some of these ellipses?
- (2) Determine the affine transforms which maps the points  $(0, 0)$ ,  $(1, 0)$  and  $(0, 1)$  to the points  $(1, 1)$ ,  $(2, 0)$  and  $(0, 0)$ .
- (3) Prove that every parabola is affine-congruent to the parabola with equation  $x = y^2$ .

Hint: You can use Euclidean transformations and assume that the parabola is in the standard form.

- (4) Ellipse  $E$  lies in triangle  $\triangle ABC$  and touches the sides  $AB$ ,  $BC$  and  $CA$  at the point  $R$ ,  $P$  and  $Q$ . Prove that

$$\frac{AR}{RB} \cdot \frac{BP}{PC} \cdot \frac{CQ}{QA} = 1.$$

Are the lines  $AP$ ,  $BQ$  and  $CR$  concurrent?