

Department of Mathematics and Statistics

Geometry

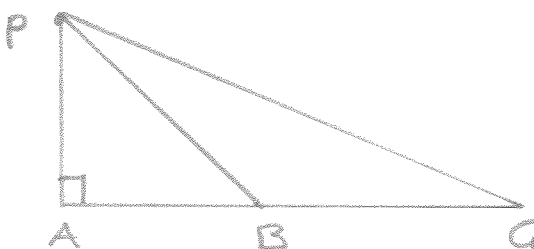
1. midterm exam

14.3.2005

- (1) **Absolute geometry.** Let l be a line and P a point not lying on it. Prove that it is possible to draw a straight line at right angles to the line l from the point P .

Instruction: You can use Euclid's postulates P1-P4 and Propositions I.1 -I.11 from Elements.

- (2) **Absolute geometry.** Let l be a line and P a point not lying on it. Drop perpendicular PA to l . Set $AB = AP$ and $BC = BP$. Prove that $\angle PCA \leq R/4$.



Instruction: You can use Euclid's postulates P1-P4, Propositions I.1 -I.28 from Elements and Saccheri-Legendre theorem.

- (3) **Hyperbolic geometry.** Let $\triangle ABC$ and $\triangle A'B'C'$ be triangles with $\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$, $AB \geq A'B'$ and $AC \geq A'C'$. Prove in hyperbolic geometry that the triangles $\triangle ABC$ and $\triangle A'B'C'$ are congruent.

Instruction: You can use Euclid's postulates P1-P4, Propositions I.1 -I.28 from Elements, hyperbolic parallel postulate and angles sum of triangles.

- (4) **Hilbert's axioms.** Let l be a line and P a point not lying on it. Prove that there exists a point $R \notin l$, which is on opposite side of l than P .

Instruction: You are allowed to use only Hilbert's axioms.