EVOLUTION AND THE THEORY OF GAMES (2-4-2009)

1. Give the definition of a Nash equilibrium and calculate all Nash equilibria (pure and mixed) in the following game:

	y_1	y_2	y_3	y_4
x_1	1 , 2	3 , 4	4 , 3	1 , 2
x_2	5 , 5	4 , 6	2 , 3	1 , 8
x_3	1 , 1	6 , 2	3 , 3	2 , 1

2. Give the definition of an evolutionarily stable strategy (ESS), and show that in the Hawk-Dove game with payoff matrix

	H	D
H	$\frac{1}{2}(V-C)$, $\frac{1}{2}(V-C)$	V , 0
D	0 , V	$\frac{1}{2}V$, $\frac{1}{2}V$

the strategy H is an ESS if $V = C$.

3. Show that the support of an ESS cannot contain a strictly dominated strategy.

4. Consider the Hawk-Dove game with a size-asymmetry between the players such that the cost C of injury inflicted by the taller player on the smaller one is greater than the cost c of injury inflicted by the smaller player on the taller one (i.e., $C > c > 0$). Give the payoff matrix and calculate all ESS solutions.

5. Consider the symmetric two-stage game $\Gamma = (\Gamma_X, \Gamma_Y)$ with payoff matrices (payoffs to the row player)

Γ_X	x_1	x_2
x_1	$V-C$	$V-C$
x_2	$\varepsilon\Gamma_Y$	$\varepsilon\Gamma_X$

Γ_Y	y_1	y_2
y_1	$-C$	$-C+\varepsilon\Gamma_X$
y_2	0	$\varepsilon\Gamma_Y$

Calculate the 4×4 payoff matrix for the full game Γ assuming that we start with the subgame Γ_X. Which of the four pure strategies (x_1, y_1), (x_1, y_2), (x_2, y_1), (x_2, y_2) is an ESS if $V > C > 0$ and which if $C > V > 0$? How does the result depend on the discounting factor ε?