

Differential equations II

Course exam 13.12.2005

No calculators or cell phones, math tables are allowed

1. Determine the general solution of the equation $y'' + 3y' + 2y = \sin x$.
2. (From homework) Determine the critical points and trajectories of the autonomous system

$$\begin{cases} x'(t) = (x - 4)(1 - y) \\ y'(t) = (x + 1)(x - 4) \end{cases},$$

and also sketch the time behavior.

3. Determine the general solution of the system $x'(t) = Ax(t)$ when A is given by

$$A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}.$$

4. Solve the initial value problem

$$x'(t) = \begin{pmatrix} 5 & 4 \\ -1 & 0 \end{pmatrix} x(t), \quad x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$