19.5.09

Final Exam

Dependence logic

For each problem, give a complete and detailed solution.

1. Show carefully that $\models \forall x_0 \forall x_1 (x_0 = x_1 \ \vee \ \neg x_0 = x_1)$.
2. Prove that the formulas $=(x_0, x_1, x_2)$ and $=(x_1, x_0, x_2)$ are logically equivalent, but the formulas $=(x_0, x_1, x_2, x_3)$ and $=(x_0, x_0, x_2, x_3)$ are not.
3. Show that, if a sentence of dependence logic is true in every finite structure with an even number of elements, then it is true in an infinite structure.
4. Give a sentence of dependence logic which is true in a graph if and only if the graph is disconnected.
5. Prove that the decision problem of dependence logic is non-arithmetical, that is, that the set of Gödel numbers of valid sentences of dependence logic is not first-order definable in the structure $(\mathbb{N}, +, \cdot, 0, 1)$.

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Dependence logic

For each problem, give a complete and detailed solution.

1. Show that the following formula is logically equivalent with a first order formula, and give the first order formula.

   $$\exists x_0 \forall x_1 (=(x_0, x_1) \wedge R x_0 x_1 x_2).$$

   You may assume that the universe has at least two elements.

2. Show that the formula $=(x_0, x_1) \vee =(x_2, x_3)$ is not first order definable.

3. Give a sentence of dependence logic which is true in a finite structure if and only if the size of the structure is of the form $3n + 1$ for some natural number n.

4. Let $M = (\mathbb{Z}, f)$, where $fz = z^2$. Show that II does not have a uniform winning strategy in the imperfect information semantic game $H(\phi)$, where ϕ is the sentence $\forall x_0 \exists x_1 (=(f x_0, x_1) \wedge x_0 = x_1)$? Why is the strategy of II of choosing $x_1 = x_0$ not uniform?

5. Give a sentence of dependence logic which is logically equivalent to the following Σ^1_1-sentence:

   $$\exists f \forall x_0 \forall x_1 \phi(x_0, x_1, f(x_0, x_1), f(x_1, x_0)),$$

   where ϕ is quantifier-free.