Todennäköisyyslaskennan kurssi, fall 2008, 1. course exam 20.10.2008

**You are not allowed to use mathematical tables in the exam.**

**1.** A con artist has three coins in his pocket. Two of them are ordinary one euro coins (one side is heads, the other one is tails), but the third one is a fake, which has tails on both sides of the coin. The con artist selects one of the three coins at random and tosses it. The result is tails. What is the probability that the con artist has chosen the fake coin?

**2.** Let $X$ have the exponential distribution with expected value $1/\lambda$, where $\lambda > 0$. Give formulas for its probability density function, distribution function, quantile function, variance and median. (You remember some of the results, and others you must calculate. Write down your calculations.) Check that the formula you give for the distribution function has the properties any distribution function must have.

**3.** Let $U$ and $V$ be independent random variables with the distributions

$$U \sim N(0,1), \quad V \sim N(1,2).$$

(In the normal distribution $N(\mu, \sigma^2)$ the first parameter is the mean and the second one the variance.) Define

$$X = U - 2V, \qquad Y = U^2 V.$$

a) Calculate the mean and variance of $X$.

b) Calculate the mean of $Y$.

c) Calculate the variance of $Y$.

**4.** Define the random variable $X$ as in the previous problem.

a) Calculate the moment generating function of $X$.

b) Recognize the distribution of $X$ based on its moment generating function.

## Formulas

$$\Gamma(t) = \int_0^\infty x^{t-1}\, \mathrm{e}^{-t}\, \mathrm{d}t, \qquad t > 0,$$

$$\Gamma(1) = 1, \quad \Gamma(\frac{1}{2}) = \sqrt{\pi}, \quad \Gamma(t+1) = t\,\Gamma(t).$$

$$B(a,b) = \int_0^1 x^{a-1}\,(1-x)^{b-1}\,\mathrm{d}x = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}, \qquad a,b > 0.$$

$$\mathrm{e}^x = \sum_{j=0}^{\infty} \frac{x^j}{j!}.$$

$$(1+x)^r = \sum_{j=0}^{\infty} \binom{r}{j} x^j, \qquad |x| < 1.$$