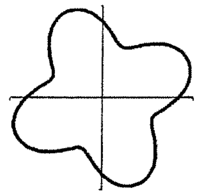
Analyysin peruskurssi
20.1.2005

In problem 3 solve either part a) or part b). If both parts are solved, then both will be graded and the better solution dropped.

1. Determine the limits:
   a. $\lim_{x\to\infty} \sqrt{x^2+x} - \sqrt{x^2-2x}$
   b. $\lim_{x\to\infty} x\sin\left(\frac{1}{x^2}\right)$
2. Determine the integrals:
   a. $\int \frac{1}{4+x^2}dx$
   b. $\int \cos^5(x)dx$
3. Solve **one** of the following two problems::
   a. Determine the area of the domain bounded by the polar curve $r^2 = 2+\sin(4\theta)$



$r^2 = 2+\sin(4\theta)$

   b. Determine the length of the curve $y = \cosh(x)$, $0 \le x \le 1$.
4. Express the integral $\int_0^{\frac{1}{2}} \frac{1}{1+x^3}dx$ as the sum of an alternating series by replacing the integrand by its Taylor series at the origin. Approximate the value of the integral by a partial sum of the series so that the error is less than 0.0001. Justify your answer.

Single Variable Calculus (Analyysin peruskurssi)
Final Exam
2007-04-12

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1. Let $f(x) = e^x - 3x$. Find the maximum and the minimum of f in the interval $[0, 1]$.

2. Let $f(x) = x^2 - 2$. Calculate successive approximations x_1, x_2 and x_3 for a root of f by Newton's method, starting with $x_0 = 1$.

3. Calculate $f''(0)$, where $f(x) = x^2 - 2^x$.

4. Show that for all natural numbers n it holds that

$$\lim_{x \to \infty} x^n e^{-x} = 0.$$

(Hint: l'Hospital's rule and induction.)

5. Define $f : \mathbb{N} \to \mathbb{R}$ with an improper integral as follows:

$$f(n) = \int_0^\infty t^n e^{-t}\, dt.$$

Show that for all $n \in \mathbb{N}$ it holds that

$$f(n + 1) = (n + 1)f(n).$$

The claim of the previous problem may be assumed true. (Hint: Integration by parts.)