

DEPARTMENT OF MATHEMATICS AND STATISTICS

Analysis I

Examination 2

Dec 13, 2007

Don't forget the course questionnaire on the web!

1. Determine

$$\lim_{x \rightarrow 3} \left( \frac{(x+1)(x-1)}{2x} + \frac{2x}{(x+1)(x-1)} \right)$$

using the theorems of the course.

2. Show using the definitions of limit and derivative that  $f'(1) = -3$  if

$$f(x) = \frac{x+1}{2x-1}$$

for all  $x > \frac{1}{2}$ .

3. Show that among the values that

$$e^{-x^2} \sin x$$

gets in the set of reals, there is a largest one. (All familiar properties of sinus and the exponential may be used in the solution.)

4. Show that

$$\frac{\sin^2 x}{x} \leq 2$$

for all  $x > 0$ . (The claim is equivalent with the inequality

$$\sin^2 x \leq 2x.$$

Of course you can use any of the formulations you want.) Hint: the mean value theorem.

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2nd midterm

11. 12. 2008

You may use all familiar properties of the trigonometrical functions and the exponential function in problems 3 and 4.

1. Determine the limit value using the theorems you studied in the course

$$\lim_{x \rightarrow 4} \frac{(x+1)(x-1)}{(2x+1)(2x-1)}.$$

2. Show, using the definition of the limit value of a function and the definition of the derivative, that  $f'(2) = \frac{1}{9}$ , if it for every  $x > 0$  holds that

$$f(x) = \frac{x}{x+1}.$$

3. Does the inequality  $e^x > 1 + \sin x$  hold for every  $x > 0$ ? Prove your assertion!

4. Show that there exists such an  $x > 0$  that

$$\frac{\sin(\frac{\pi}{2}e^x)}{x^2+1} = \frac{1}{10^{100}}.$$