

DEPARTMENT OF MATHEMATICS AND STATISTICS

Analysis II

Final exam

19. 12. 2007

1. Prove that $f(x) = |3x - 2|$ is (Riemann-)integrable on $[0, 1]$.

2. Determine

$$\int_1^e x(\ln x)^2 dx.$$

3. Does the series

$$\sum_{k=1}^{\infty} \frac{1}{2k-1}$$

converge. Prove your assertion.

4. Prove that $\sum x^k$ does not converge uniformly on $]0, 1[$.

5. Compute for $f(x) = e^x$ the second order Taylor polynomial $T_2(x; 1)$ centered around $x_0 = 1$ and use it to determine

$$\lim_{x \rightarrow 1} \frac{e^x - e x}{(x-1)^2}.$$

Prove your assertion.

DEPARTMENT OF MATHEMATICS AND STATISTICS
Analysis II
Final exam
11. 6. 2009

1. Compute

$$\int_1^2 x e^x dx.$$

2. Does the improper integral

$$\int_0^{\infty} \frac{2x}{(x^2 + 1)^2} dx$$

converge?

3. Does the series

$$\sum_{k=1}^{\infty} \frac{k+1}{k^2}$$

converge? Explain your answer carefully!

4. Consider the function $f_n : \mathbb{R} \rightarrow \mathbb{R}$, where

$$f_n(x) = x^{2009} + \frac{1}{n^2 x^2 + n}.$$

Prove that the sequence (f_n) converges uniformly on the whole of \mathbb{R} .

5. Assume about the function f that the second derivative f'' is continuous in the interval $]0, 2[$ and that $f(1) = 1$, $f'(1) = 2$ and $f''(1) = 84$. Form the Taylor polynomial $T_2(x; 1)$ of the function f and determine

$$\lim_{x \rightarrow 1} \frac{f(x) - 2x + 1}{x^2 - 2x + 1}$$

using $T_2(x; 1)$ and Taylor's formula. Explain your answer carefully!