DEPARTMENT OF MATHEMATICS AND STATISTICS
Calculus II, 2008
2. course exam

PLEASE REMEMBER TO ANSWER THE COURSE EVALUATION

The assignments are ordered according to theme.

1. Is the series

$$\sum_{k=1}^{\infty} \frac{k}{3k^2+5}$$

convergent or divergent?

2. Consider the functions $f_n :]0,1[\to \mathbb{R}$, defined through

$$f_n(x) = \frac{1}{nx},$$

where $n = 1, 2, 3, \ldots$ Does the sequence (f_n) converge pointwise? Does it converge uniformly?

3. Find out the interval and radius of convergence for the series

$$\sum_{k=0}^{\infty} (\frac{k+1}{k})^{k^2} (x-7)^k.$$

4. Calculate the limit

$$\lim_{x \to 0} \frac{e^{\sin x} - \sin x - 1}{\sin^2 x}$$

by using a suitable Taylor polynomial for e^t, where $x_0 = 0$.

DEPARTMENT OF MATHEMATICS AND STATISTICS
Analyysi II

2nd midterm 4.5.2009

The tasks are ordered by subject. Show the working that led to your conclusions.

1 Does the series

$$\sum_{k=1}^{\infty} \frac{5k}{3k^2 - 2}$$

converge?

2 Does the function sequence $f_1, f_2, \ldots$ converge uniformly in the whole set of real numbers if we for every n have

$$f_n(x) = \frac{1}{n} \sin(x^n)?$$

3 We assume that the radius of convergence for the power series $\sum_{k=0}^{\infty} a_k x^k$ is 7. We assume that $|b_k| < |a_k|$ for every k and that the radius of convergence for the series $\sum_{k=0}^{\infty} b_k x^k$ is R. Show that $R \geq 7$.

4 (a) Construct the Taylor polynomial $\mathrm{T}_2(x; \frac{\pi}{4})$ for the function

$$f(x) = \sin x + \cos x.$$

(b) Use the previous part to calculate the limit value

$$\lim_{x \to \frac{\pi}{4}} \frac{(4x - \pi)^2}{\sin x + \cos x - \sqrt{2}}.$$