

DEPARTMENT OF MATHEMATICS AND STATISTICS

Calculus II

Course exam 1

28. 2. 2008

1. Calculate

$$\int_0^1 x^2 \sin\left(\frac{\pi x^3}{4}\right) dx.$$

2. Choose ONE of the following assignments

(a) Calculate

$$\int_1^e \sin(\ln x) dx.$$

(b) Consider the function $f : [0, 1] \rightarrow \mathbb{R}$, for which $f(x) = 0$ when $x \neq 1$ and $f(1) = 3$. Give an example of a subdivision D of the interval $[0, 1]$ for which $S_D - s_D < 2^{-100}$. Motivate!

3. Is the integral

$$\int_1^\infty \frac{x^2 + 1}{x^4 + 1} dx$$

convergent or divergent?

4. We define a function $f : \mathbb{R} \rightarrow \mathbb{R}$ through

$$f(x) = \sin(x^2).$$

Is f uniformly continuous in the set of all real numbers?

DEPARTMENT OF MATHEMATICS AND STATISTICS
Analyysi II

1st midterm 26.2.2009

1. Calculate

$$\int_0^{\sqrt[3]{\pi/3}} x^2 \cos(x^3) dx.$$

2. Calculate

$$\int_e^{e^2} \frac{1}{x \ln x} dx.$$

3. Show that the improper integral

$$\int_1^{\infty} \frac{\sqrt{x}}{2x-1} dx$$

diverges.

4. (a) (2 points) Explain the Riemann condition. (You do not need to prove it.)

(b) (4 points) We define the function $f : \mathbb{R} \rightarrow \mathbb{R}$ by setting $f(0) = 0$ and $f(x) = \sin \frac{1}{x}$ when $x \neq 0$. Show (for example using the Riemann condition) that the function f is (Riemann-)integrable on the interval $[0, 1]$.