DEPARTMENT OF MATHEMATICS AND STATISTICS
Calculus I.2
Mid-term examination 1
10. 3. 2005

1. We study a bounded function f defined in the interval $[0, 2]$, and the subdivisions D_1 and D_2 of the interval $[0, 2]$. Assume that $7 \leq S_{D_1} < 7 + 2^{-17}$ and that $7 - 2^{-17} \leq s_{D_2} < 7$. Give an example of a subdivision D, for which $S_D - s_D < 2^{-16}$. Motivate your answer!

2. Compute
$$f(x) = \int_0^{\pi/3} \sin(\pi \cos x) \sin x \, dx.$$

3. For every x, let
$$f(x) = \int_a^{e^x} \sin t \, dt - \int_b^x \sin(e^t) e^t \, dt$$
Also assume that $f(0) = 1$. Evaluate $f(2005)$.

4. Assume that the function f is continuously differentiable everywhere (i.e. the derivative is continuous). Assume that $f(0) = 1 = f(1)$. Compute
$$\int_0^1 f(x) f'(x) e^{f(x)} \, dx.$$
Tip: partial integration.

DEPARTMENT OF MATHEMATICS AND STATISTICS
Analysis II
Course examination
Mar 2, 2006

1. Consider the function $f : [0, 3] \to \mathbb{R}$ such that $f(x) = 1$ when $0 \leq x \leq 1$, $f(x) = 2$ when $1 < x < 2$, and $f(x) = 3$ when $2 \leq x \leq 3$. Give an example of a subdivision D of the interval $[0, 3]$ such that $S_D - s_D < 2^{-100}$. Justify your answer!

2. Calculate
$$f(x) = \int_1^e \frac{\ln x}{x}\, dx.$$

3. Define the function $f :]\pi, 3\pi[\to \mathbb{R}$ by the condition
$$f(x) = \int_0^{\sin x} e^{\cos t}\, dt.$$

   (a) Why is f differentiable in the interval $]\pi, 3\pi[$? Justify your answer!

   (b) Differentiate f.

4. Assume that $f :]-1, 2[\to \mathbb{R}$ is strictly increasing and differentiable, and that f' is continuous in the entire interval. Assume also that $f(0) = 0$ and $f(1) = 1$. Show that
$$\int_0^1 (f^{-1}(x) - xf'(x))\, dx = 0.$$

   Here f^{-1} denotes the inverse function. Hint: think about the difference of two integrals and use the substitution $x = f(t)$ in one of them.