DEPARTMENT OF MATHEMATICS AND STATISTICS
Analysis I
Final exam
18. 12. 2007

1. Suppose that $|x-7|<3$ and $|2x-3|<9$. Show that $|x-5|<1$.

2. Find

$$\lim_{x\to 2}\frac{(2x+3)(4x^2-5)}{6x^3+7}.$$

and prove your assertion.

3. Suppose $\lim_{x\to 1+} f(x)=2$ and $\lim_{x\to 1+} g(x)=\infty$. Prove that $\lim_{x\to 1+}(f(x)+g(x))=\infty$.

4. Suppose that $f:]-1,1[\to\mathbb{R}$ has the properties $f(0)=0$ and $f'(0)=1$. Prove that there is such an $h>0$ that for all $x\in]-h,0[$ the inequalities $(1+7^{-7})x<f(x)<(1-7^{-7})x$ hold.

5. Assume that $f:]x_0-h,x_0+h[\to\mathbb{R}$ is continuous in $]x_0-h,x_0+h[$ and differentible in $]x_0-h,x_0[$ and in $]x_0,x_0+h[$. Suppose moreover that $\lim_{x\to x_0-} f'(x)=\lim_{x\to x_0+} f'(x)=A\in\mathbb{R}$. Prove that $f$ is differentiable at $x_0$ and $f'(x_0)=A$.

NOTATION: Above $]a,b[$ means the open interval $\{x \mid a<x<b\}$.