Matematiikan laitos
Algebrallinen topologia II
Loppukoe
3.3.2009

1. Prove the Fundamental Theorem of Algebra.

2. Prove the following result. Let X be a topological space and let $\{X_a, i_a^b\}_\Lambda$ be a direct system of subsets of X satisfying the following condition. If $C \subset X$ is a compact subset of X, then there exists $b \in \Lambda$ such that $C \subset X_b$. Then

$$\varinjlim_{a \in \Lambda} \{H_n(X_a)\} \cong H_n(X), \text{ for every } \ n \geq 0.$$

3. Prove the following result. Let $B \subset S^n$ be a subset of S^n such that B is homeomorphic to S^k, where $0 \leq k \leq n-1$. Then

$$\tilde{H}_j(S^n - B) \cong \begin{cases} 0, & j \leq n-k-1, \\ \mathbb{Z}, & j = n-k-1. \end{cases}$$

4. State and prove the Jordan–Brouwer separation theorem.

5. Formulate and prove the theorem of invariance of domain.

Matematiikan laitos
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1. Prove the Fundamental Theorem of Algebra.

2. Prove the following result. Let X be a topological space and let $\{X_a, i_a^b\}_\Lambda$, $a \in \Lambda$, be a direct system of subsets of X satisfying the following condition. If $C \subset X$ is a compact subset of X, then there exists $b \in \Lambda$ such that $C \subset X_b$. Then

$$\varinjlim_{a \in \Lambda} \{H_n(X_a)\} \cong H_n(X), \text{ for every } n \geq 0.$$

3. Prove the following result. Let $B \subset S^n$ be a subset of S^n such that B is homeomorphic to S^k, where $0 \leq k \leq n-1$. Then

$$\tilde{H}_j(S^n - B) \cong \begin{cases} 0, & j \neq n-k-1, \\ \mathbb{Z}, & j = n-k-1. \end{cases}$$

4. State and prove the Jordan–Brouwer separation theorem.

5. Formulate and prove the theorem of invariance of domain.