



The time to use in this exam is **1 hour 55 minutes**. Take care yourself that you return your solutions to supervisors of the exam not later than 1 hour and 55 minutes after the exam starts.

1. Use induction to prove that the number $10^{n+1} + 3 \cdot 10^n + 5$ is divisible by the number 9 for every $n \in \mathbb{N}$.
2. Let X be a set and (Y, \leq) be a partially ordered set and $f: X \rightarrow Y$ be an injection. Define a relation \leq in X by

$$x \leq y \iff f(x) \leq f(y).$$

Show that (X, \leq) is a partially ordered set

3. Let $G = \{x \in \mathbb{R} \mid x > 1\}$ and define

$$x \circ y = xy - x - y + 2, \text{ if } x, y \in G.$$

Prove that (G, \circ) is an Abelian group.

4. Let G be an Abelian group and $m \in \mathbb{Z}$. Show that

$$H = \{a \in G \mid a^m = 1_G\}$$

is a subgroup of G .