

Matematiikan ja tilastotieteen laitos

Algebra I

1. Midterm exam. March 1, 2006

1. Determine the greatest common divisor of 1479 and 272 using Euclid's Algorithm.
2. Let $t_n = \frac{n(n+1)}{2}$, when $n \in \mathbf{N}^* = \{1, 2, 3, \dots\}$. Using mathematical induction, show that

$$t_1 + t_2 + \dots + t_n = \frac{n(n+1)(n+2)}{6}$$

for all $n \in \mathbf{N}^*$.

3. Investigate whether (\mathbf{Z}, \cdot) is a group.
4. Remember that when $n \in \mathbf{N}^*$, then the multiplicative group (\mathbf{Z}_n^*, \cdot) consists of the set $\mathbf{Z}_n^* = \{\bar{a} \in \mathbf{Z}_n \mid \gcd(a, n) = 1\}$, with the multiplication of \mathbf{Z}_n as the operation. Determine the set \mathbf{Z}_{14}^* , the multiplication table of the group $(\mathbf{Z}_{14}^*, \cdot)$, and (using the table) the inverse elements of all the $\varphi(14) = 6$ elements of this group.