EVOLUTION AND THE THEORY OF GAMES

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Exercises 28-09-2016

7. Prove that if x and y with $x \neq y$ are both ESS of the same game, then $\mathrm{supp}(x) \not\subset \mathrm{supp}(y)$ and $\mathrm{supp}(y) \not\subset \mathrm{supp}(x)$.

8. As a variation of the Hawk-Dove game, and instead of the H and D strategies, consider the strategies R and B where R (=retaliator) always starts like D but immediately switches to H when its opponent plays H, and where B (=bully) always starts as H, but immediately switches to D if the other player plays H. Give the payoff matrix of the Retaliator-Bully game, and calculate all ESSs, mixed as well as pure. (Note: the payoffs from a B×B contest are not uniquely specified in the above description. What payoffs to use here I leave up to you, but be prepared to defend your choice as a model of reality.)

9. Prove the following generalisation of the Bishop-Cannings theorem: if x is a mixed ESS that can be represented by a probability density f over a continuum of pure strategies and, moreover, the payoff $E(\cdot, x)$ is a continuous function on the set of pure strategies, then $E(\xi, x) = E(x, x)$ for every pure strategy ξ in the support of x. (Note: if you can, you may want to prove an even more general version: if x is an ESS (mixed or pure), then $E(\xi, x) = E(x, x)$ for x-almost every pure strategy ξ.)

10. (a) Use the War of Attrition (WoA) to model a Hawk×Hawk contest where each pure strategy $c \geq 0$ is the maximum cost of injury a player is prepared to endure before giving up. Calculate the ESS as well as the expected payoff at the ESS. **(b)** Give all ESSs of the "double-corrected" Hawk-Dove payoff matrix

	H	D
H	E(H,H)	R
D	0	0

where E(H,H) is the expected payoff at the ESS of the WoA in part (a) of this exercise.