Finite model theory

Problems 7

Tuesday 1.11.2016

1. Let $k \geq 1$. Construct a sentence φ of second-order logic such that for all finite structures $\mathfrak{A}$:

$$\mathfrak{A} \models \varphi \Leftrightarrow |\mathrm{Dom}(\mathfrak{A})| \text{ is divisible by } k.$$

2. Construct a sentence φ of monadic second-order logic such that for all finite ordered structures $\mathfrak{A}$:

$$\mathfrak{A} \models \varphi \Leftrightarrow |\mathrm{Dom}(\mathfrak{A})| \text{ is even.}$$

3. A graph $\mathbb{G}$ is bipartite if there is $U \subseteq \mathrm{Dom}(\mathbb{G})$ such that all edges of $\mathbb{G}$ are between elements of U and $\mathrm{Dom}(\mathbb{G}) \setminus U$. Construct a sentence φ of monadic second-order logic such that for all finite graphs $\mathbb{G}$:

$$\mathbb{G} \models \varphi \Leftrightarrow \mathbb{G} \text{ is bipartite.}$$

4. Let $L \subseteq \Sigma^*$ be recognized by a finite automaton. Show that there exists $m \in \mathbb{N}$ such that for all $s \in \Sigma^*$ with $|s| \geq m$, s can be written as uvw, where $u, v, w \in \Sigma^*$, $v \neq \lambda$, $|uv| \leq m$, and for all $k \in \mathbb{N}$

$$uvw \in L \Leftrightarrow uv^k w \in L.$$

5. Let Σ have at least two letters. Show that the language L consisting of all palindromes w over Σ cannot be recognized by a finite automaton. Recall that a word $w = \alpha_0 \ldots \alpha_n$ is a palindrome if $\alpha_i = \alpha_{n-i}$ for all i.

6. Let $\Sigma = \{a, b\}$, and $L = \{w \in \Sigma^+ \mid w \text{ has more occurrences of } a \text{ than } b\}$. Show that L cannot be recognized by a finite automaton.