Finite model theory

Problems 6

Tuesday 18.10.2016

1. Let $\Sigma = \{a, b, c\}$. Construct finite automata recognizing the following languages:

1. $L_0 = \{w \in \Sigma^+ \mid |w| = 0 \text{ mod } 4\}$
2. $L_1 = \{w = \alpha_0 ... \alpha_j \in \Sigma^+ \mid \alpha_i \neq \alpha_{i+1} \text{ for all } 0 \leq i \leq j-1\}$
3. $L_2 = \{w \in \Sigma^+ \mid w = a^k b^l c^t \text{ for some } k, l, t \geq 1\}$

2. Let $L \subseteq \Sigma^*$ be a finite language. Show that L can be recognized by a finite automaton.

3. Show that the languages L_1 and L_2 above can be defined in first-order logic.

4. Show that the language $L_0 \setminus \{\lambda\}$ cannot be defined in first-order logic.

5. Let $\mathfrak{A}$, $\mathfrak{A}'$, $\mathfrak{B}$, and $\mathfrak{B}'$ be finite ordered relational τ-models such that $\mathfrak{A} \cong_k \mathfrak{A}'$ and $\mathfrak{B} \cong_k \mathfrak{B}'$. Show that for the ordered sums the following holds:

$$\mathfrak{A} \triangleleft\!\!\!+ \mathfrak{B} \cong_k \mathfrak{A}' \triangleleft\!\!\!+ \mathfrak{B}'.$$

6. Let $\Sigma = \{a, b\}$, and $L = \{w \in \Sigma^+ \mid w \text{ has more occurrences of } a \text{ than } b\}$. Show that L cannot be defined in first-order logic.