

Finite model theory
 Problems 4
 Tuesday 4.10.2016

1. Let \mathfrak{G} and \mathfrak{G}' be graphs of cardinality 4. Show that Spoiler has a winning strategy in the game $\text{EF}_2(\mathfrak{G}, \mathfrak{G}')$

- a) if $\mathfrak{G} \not\cong \mathfrak{G}'$ and \mathfrak{G} has exactly 0 or 6 edges.
- b) if \mathfrak{G} has a node with no edges and \mathfrak{G}' does not have such a node.

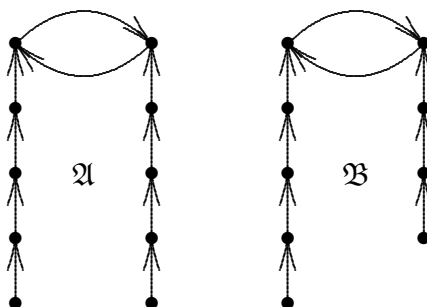
2. Let \mathfrak{A} , \mathfrak{B} , and \mathfrak{C} be τ -models and $k \in \mathbb{N}$. Show that if $\mathfrak{A} \cong_k \mathfrak{B}$ and $\mathfrak{B} \cong_k \mathfrak{C}$, then $\mathfrak{A} \cong_k \mathfrak{C}$.

3. Let $n \in \mathbb{N}$ and $A_n = \{m \in \mathbb{N} : m|n\}$, that is, A_n contains the positive factors of n . Define $\{R\}$ -models (R a binary relation symbol) \mathfrak{A} and \mathfrak{B} as follows:

- $\text{Dom}(\mathfrak{A}) = A_{24}$, and $\text{Dom}(\mathfrak{B}) = A_{36}$,
- $R^{\mathfrak{A}} = \{(k, s) \in (A_{24})^2 : k|s\}$, and $R^{\mathfrak{B}} = \{(k, s) \in (A_{36})^2 : k|s\}$.

Determine the greatest $k \in \mathbb{N}$ such that $\mathfrak{A} \cong_k \mathfrak{B}$.

4. Let R be a binary relation and let \mathfrak{A} and \mathfrak{B} be the following $\{R\}$ -models. Determine the smallest k such that there exists a formula φ of quantifier rank k that separates \mathfrak{A} and \mathfrak{B} (i.e., φ holds in exactly one of the models).



5. Let \mathfrak{A} and \mathfrak{B} be τ -models, $p \in \text{Part}(\mathfrak{A}, \mathfrak{B})$, and $I \subseteq \text{Part}(\mathfrak{A}, \mathfrak{B})$. Assume that every $g \in I$ preserves a τ -formula $\varphi(\vec{x}, y)$ and that p satisfies the forth condition with respect to I . Show that p preserves the formula $\exists y \varphi(\vec{x}, y)$.

6. Let $\tau = \{U, V\}$ where U and V are unary relation symbols. Show that there is no τ -sentence φ of first-order logic such that for all finite τ -models \mathfrak{A} holds:

$$\mathfrak{A} \models \varphi \Leftrightarrow |U^{\mathfrak{A}}| = |V^{\mathfrak{A}}|.$$