Finite model theory

Problems 3
Tuesday 27.9.2016

1. Show that for every $\mathcal{L}_{\infty,\omega}$-sentence φ over a finite τ there is a $\mathcal{L}_{\omega_1,\omega}$-sentence ψ such that for all finite τ-models $\mathfrak{A}$

$$\mathfrak{A} \models \varphi \Leftrightarrow \mathfrak{A} \models \psi.$$

2. Let τ be finite vocabulary, and $\mathfrak{A}$ and $\mathfrak{B}$ a finite τ-models. Show that the following are equivalent:

1. $\mathfrak{A}$ and $\mathfrak{B}$ satisfy the same sentences of FO^k,
2. $\mathfrak{A}$ and $\mathfrak{B}$ satisfy the same sentences of $\mathcal{L}^k_{\infty,\omega}$.

Hint: Show that the claim of Exercise 1 holds for the logics $\mathcal{L}_{\infty,\omega}$ and FO when equivalence with respect to all finite models is replaced by equivalence with respect to $\mathfrak{A}$ and $\mathfrak{B}$.

3. Let $n > 0$. Construct an FO^2-formula $\psi_n(x)$ such that for any finite linear order $\mathfrak{A}$ the following holds: $\mathfrak{A} \models \psi_n[a/x]$ iff a has exactly n predecessors with respect to $\leq^{\mathfrak{A}}$.

4. Let K be a class of finite ordered τ-models, where τ ($\leq \in \tau$) is a finite vocabulary consisting of relation symbols of arity at most 2. Show that K can be axiomatized by a sentence of $\mathcal{L}^2_{\infty,\omega}$.

5. Let $\mathfrak{A}$ and $\mathfrak{B}$ be finite linear orders such that $|Dom(\mathfrak{A})| \neq |Dom(\mathfrak{B})|$ and $|Dom(\mathfrak{A})| < 2^{k-1}$. Show that the spoiler has a winning strategy in $EF_k(\mathfrak{A}, \mathfrak{B})$.

6. Determine the greatest k such that $\mathbb{G} \cong_k \mathbb{G}'$ holds.