Finite model theory

Problems 11
Tuesday 29.11.2016

1. Let $\tau = \{<\}$ where $<$ is an arbitrary binary relation symbol. Show that for all $n \geq 1$ there is a satisfiable $\mathrm{FO}^2_=[\tau]$-sentence having only models of cardinality $\geq n$. (Note that the class of orderings is not axiomatizable in $\mathrm{FO}^2_=$.)

2. $\Sigma^1_1(FO^1_=)[\tau]$ is the fragment of $ESO[\tau]$ in which the first-order part is in $FO^1_=$ (i.e. of the form $\exists R_1 \ldots \exists R_n \phi$, where $\phi \in FO^1_=[\tau \cup \{R_1, \ldots, R_n\}]$).
a) Show that $\Sigma^1_1(FO^1_=)$ has the finite model property.
b) Show that $FO^3_=$ does not have the finite model property.
c) Does $FOC^2_=$ have the finite model property?

FOC (first-order logic with counting) extends FO by the following quantifier for each $n \in \mathbb{N}$: $\exists_{\geq n} x\, \varphi(x)$,

$$\mathfrak{A} \models \exists_{\geq n} x\, \varphi(x) \text{ iff } \mathfrak{A} \models \varphi(a) \text{ for at least } n \text{ distinct } a \in A.$$

$FOC^2_=$ is the two-variable fragment of FOC.