Finite model theory

Problems 1

Tuesday 13.9.2016

1. Give examples of the following types of binary relations R:

1. R is reflexive, symmetric, but not transitive.
2. R is a partial-order but not a linear-order.

2. Let X be a set of cardinality n, and $k \in \mathbb{N}$. Determine the number of k-ary relations over X. How many of those are symmetric?

3. Let τ be a finite vocabulary. Show that the number of non-isomorphic τ-models of cardinality n is bounded by $2^{p(n)}$, where $p(x)$ is a polynomial function.

4. Let f be a homomorphism from $\mathfrak{A}$ to $\mathfrak{B}$, and g a homomorphism from $\mathfrak{B}$ to $\mathfrak{C}$. Show that $g \circ f$ is a homomorphism from $\mathfrak{A}$ to $\mathfrak{C}$.

5. Let $\mathfrak{A}$ and $\mathfrak{B}$ be $\{f\}$-models. Let $R_f^{\mathfrak{A}}$ and $R_f^{\mathfrak{B}}$ denote the graphs of the functions $f^{\mathfrak{A}}$ and $f^{\mathfrak{B}}$, that is

$$R_f^{\mathfrak{A}} = \{(\overline{a}, f^{\mathfrak{A}}(a)) \ : \overline{a} \in \mathrm{Dom}(\mathfrak{A})^{ar(f)}\}.$$

Let $h : \mathrm{Dom}(\mathfrak{A}) \to \mathrm{Dom}(\mathfrak{B})$ be a function. Show that h is a homomorphism from $\mathfrak{A}$ and $\mathfrak{B}$ if and only if h is a homorphism from $\mathfrak{A}^*$ to $\mathfrak{B}^*$, where $\mathfrak{A}^*$ ($\mathfrak{B}^*$) is the $\{P\}$-model such that $P^{\mathfrak{A}^*} = R_f^{\mathfrak{A}}$ ($P^{\mathfrak{B}^*} = R_f^{\mathfrak{B}}$).

6. Let $\mathbb{G} = (V, E)$ be a graph of cardinality at least 6. Show that there exists $a, b, c \in V$ such that either $\{(a, b), (b, c), (c, a)\} \subseteq E$ or $\{(a, b), (b, c), (c, a)\} \subseteq E^c$, where $E^c = V^2 - E$.