JAKAUMA	PTNF / TF $f_X(x)$	$\mathbb{E}X$	var X	$M(t)$
$X \sim Bin(n,p)$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$	$(pe^t+1-p)^n$
$X \sim Bernoulli(p)$	$\Leftrightarrow X \sim Bin(1,p)$			
$X \sim Geom(p)$	$p(1-p)^x$, $x=0,1,\ldots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$	$p\cdot(1-(1-p)e^t)^{-1}$, $t<\ln(\frac{1}{1-p})$
$X \sim Poi(\theta)$, $\theta>0$	$e^{-\theta}\frac{\theta^x}{x!}$, -"-	θ	θ	$\exp(\theta(e^t-1))$
$X \sim U(a,b)$	$\frac{1}{b-a}\mathbb{1}\{a<x<b\}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	
$X \sim Exp(\lambda)$, $\lambda>0$	$\lambda e^{-\lambda x}\mathbb{1}\{x>0\}$	$1/\lambda$	$1/\lambda^2$	$\lambda/(\lambda-t)$, $t<\lambda$
$X \sim N(\mu,\sigma^2)$	$(2\pi\sigma^2)^{-1/2}\exp(-\frac{(x-\mu)^2}{2\sigma^2})$	μ	σ^2	$\exp(\mu t+\frac{1}{2}\sigma^2t^2)$

MARKOVIN EY: $X \geq 0$, $\mathbb{E}X<\infty \Rightarrow \mathbb{P}(X\geq a) \leq a^{-1}\mathbb{E}X$, $\forall a>0$

TŠEBYŠEVIN EY: $\mathbb{E}X=\mu$, var $X=\sigma^2<\infty \Rightarrow \mathbb{P}(|X-\mu|>t) \leq t^{-2}\sigma^2$ $\forall t>0$

KONVEKSIT FUNKTIOT: , f' kasvava (jos kerran jvasti derivoituva)

$f''\geq 0$ (-"- kahdesti -"-)

↪ TARKKA MÄÄR. $g(\lambda x+(1-\lambda)y) \leq \lambda g(x)+(1-\lambda)g(y)$ $\forall \lambda\in[0,1]$, $\forall x,y\in I$

JENSENIN EY: $g(\mathbb{E}X) \leq \mathbb{E}g(X)$

jos · g konveksi, · $X\in I$ tn:llä 1, · $\mathbb{E}X$ ja $\mathbb{E}g(X)$ olemassa

HÖLDERIN EY, CAUCHYN-SCHWARZIN EY:

$\mathbb{E}|XY| \leq \|X\|_p\|Y\|_q$ ← kun $p=q=2$ $\Rightarrow |\mathrm{corr}(X,Y)|\leq 1$

kun $\frac{1}{p}+\frac{1}{q}=1$, $\|X\|_p=(\mathbb{E}|X|^p)^{1/p}$

$\mathrm{corr}(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sqrt{\mathrm{var}\,X}\sqrt{\mathrm{var}\,Y}}$, $\mathrm{cov}(X,Y) = \mathbb{E}((X-\mathbb{E}X)(Y-\mathbb{E}Y))$

SATUNN. VEKTORIT (SV) $\underset{\sim}{X}=(X_1,\ldots,X_n)$

$\underset{\sim}{X}$ diskreetti ($\mathbb{P}(\underset{\sim}{X}\in A)=\sum_{\underset{\sim}{x}\in A} f_{\underset{\sim}{X}}(\underset{\sim}{x})$, $\underset{\sim}{x}=(x_1,\ldots,x_n)$

$f_{\underset{\sim}{X}}(\underset{\sim}{x})=\mathbb{P}(\underset{\sim}{X}=\underset{\sim}{x}) = \mathbb{P}(X_1=x_1,\ldots,X_n=x_n)$)

↓ JAKAUMAN MÄÄRÄÄ SV:N PTNF

SV:N PTNF

YPTNF

$\underset{\sim}{X}$ jva (eli $(X_1,\ldots,X_n)$:llä jva yht. jakauma)

↓ JAK. MÄÄRÄÄ SV:N TF

$$\Big(\mathbb{P}(\underset{\sim}{X}\in A)=\int_A f_{\underset{\sim}{X}}(\underset{\sim}{x})d\underset{\sim}{x} = \int_{\mathbb{R}^n}\mathbb{1}_A(\underset{\sim}{x})f_{\underset{\sim}{X}}(\underset{\sim}{x})d\underset{\sim}{x}$$

$$\overset{\text{FUBINI}}{=} \int_{-\infty}^{\infty}dx_1\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}dx_n\,\mathbb{1}_A(x_1,\ldots,x_n)f_{\underset{\sim}{X}}(x_1,\ldots,x_n)\Big)$$

EHDOLLISET JAKAUMAT, KERTOLASKUSÄÄNTÖ, BAYESIN SÄÄNTÖ, MARGINALISOINTI

$\underset{\sim}{X} = (X_1, \dots, X_n)$, $\underset{\sim}{Y} = (Y_1, \dots, Y_m)$

$$\boxed{f_{\underset{\sim}{X},\underset{\sim}{Y}}(\underset{\sim}{x},\underset{\sim}{y}) = f_{\underset{\sim}{X}}(\underset{\sim}{x}) f_{\underset{\sim}{Y}|\underset{\sim}{X}}(\underset{\sim}{y}|\underset{\sim}{x})}$$

voimassa kun $(\underset{\sim}{X}, \underset{\sim}{Y})$ jva
–||– disk
$\underset{\sim}{X}$ disk, $\underset{\sim}{Y}$ jva

$$f_{\underset{\sim}{Y}|\underset{\sim}{X}}(\underset{\sim}{y}|\underset{\sim}{x}) = \begin{cases} \dfrac{f_{\underset{\sim}{X},\underset{\sim}{Y}}(\underset{\sim}{x},\underset{\sim}{y})}{f_{\underset{\sim}{X}}(\underset{\sim}{x})}, & f_{\underset{\sim}{X}}(\underset{\sim}{x}) > 0 \\ 0 & \text{muul.} \end{cases}$$

· f tiheys → voidaan sallia diskreetti osa ja osa jolla jva tn. jak

MARGINALISOINTI: $f_{\underset{\sim}{X}}(\underset{\sim}{x}) = \int f_{\underset{\sim}{X},\underset{\sim}{Y}}(\underset{\sim}{x},\underset{\sim}{y})\,d\underset{\sim}{y}$

↳ summataan disk. integroidaan jva komponenttien yli

$$= \begin{cases} \sum_{\underset{\sim}{y}} f_{\underset{\sim}{X},\underset{\sim}{Y}}(\underset{\sim}{x},\underset{\sim}{y}) & \text{kun } \underset{\sim}{Y} \text{ disk.} \\ \int f_{\underset{\sim}{X},\underset{\sim}{Y}}(\underset{\sim}{x},\underset{\sim}{y}) & \text{kun } \underset{\sim}{Y} \text{ jva} \\ \sum\int \dots & \text{kun } \underset{\sim}{Y}\text{:llä sekä disk. että jva komp.} \end{cases}$$

· $\underset{\sim}{X} \perp\!\!\!\perp \underset{\sim}{Y} \Rightarrow H(\underset{\sim}{X}) \perp\!\!\!\perp G(\underset{\sim}{Y})$ tiheys

$\mathbb{E}G(\underset{\sim}{Y})H(\underset{\sim}{X}) = \mathbb{E}G(\underset{\sim}{Y})\mathbb{E}H(\underset{\sim}{X})$ ($\perp\!\!\!\perp$)

$\mathbb{E}\underset{\sim}{X} = (\mathbb{E}X_1, \dots, \mathbb{E}X_n)$

$\mathbb{E}H(\underset{\sim}{X}) = (\mathbb{E}H_{ij}(\underset{\sim}{X}))_{ij}$

(TTL) $$\mathbb{E}g(\underset{\sim}{X},\underset{\sim}{Y}) = \begin{cases} \sum_{\underset{\sim}{x},\underset{\sim}{y}} g(\underset{\sim}{x},\underset{\sim}{y}) f_{\underset{\sim}{X},\underset{\sim}{Y}}(\underset{\sim}{x},\underset{\sim}{y}) & (\underset{\sim}{X},\underset{\sim}{Y}) \text{ disk} \\ \iint g(\underset{\sim}{x},\underset{\sim}{y}) f_{\underset{\sim}{X},\underset{\sim}{Y}}(\underset{\sim}{x},\underset{\sim}{y})\,d\underset{\sim}{x}\,d\underset{\sim}{y} & (\underset{\sim}{X},\underset{\sim}{Y}) \text{ jva} \\ \sum\int \quad \text{–||–} \end{cases}$$

↑ yli disk

$\mathbb{E}(\underset{\sim}{Z}^T) = (\mathbb{E}\underset{\sim}{Z})^T$

$\mathbb{E}(A\underset{\sim}{Z}B + C) = A(\mathbb{E}\underset{\sim}{Z})B + C$

EHD. ODOTUSARVO (VRT) EHDOLLA SUIN ARVO $\underset{\sim}{X} = \underset{\sim}{x}$:

kuin (TTL) yllä mutta ehd. tiheys:n — ehd. tiheys (voi olla disk)

$$\mathbb{E}(g(\underset{\sim}{X},\underset{\sim}{Y})|\underset{\sim}{X}=\underset{\sim}{x}) = \int g(\underset{\sim}{x},\underset{\sim}{y}) f_{\underset{\sim}{Y}|\underset{\sim}{X}}(\underset{\sim}{y}|\underset{\sim}{x})\,d\underset{\sim}{y}$$

$$\mathrm{var}(g(\underset{\sim}{X},\underset{\sim}{Y})|\underset{\sim}{X}=\underset{\sim}{x}) = m(\underset{\sim}{x})$$

$= v(\underset{\sim}{x})$; määr. kuten varianssi $f_{\underset{\sim}{Y}|\underset{\sim}{X}}$:llä

EHD. ODOTUSARVO (VRT) EHD. SU $\underset{\sim}{X}$: $\mathbb{E}(g(\underset{\sim}{X},\underset{\sim}{Y})|\underset{\sim}{X}) = m(\underset{\sim}{X})$

$\mathrm{var}(g(\underset{\sim}{X},\underset{\sim}{Y})|\underset{\sim}{X}) = v(\underset{\sim}{X})$; eli numero $\underset{\sim}{X}$:stä

OMINAISUUKSIA: · $\mathbb{E}g(\underset{\sim}{X},\underset{\sim}{Y}) = \mathbb{E}\,\mathbb{E}(g(\underset{\sim}{X},\underset{\sim}{Y})|\underset{\sim}{X})$

· $\mathrm{var}\,g(\underset{\sim}{X},\underset{\sim}{Y}) = \mathbb{E}\,\mathrm{var}(g(\underset{\sim}{X},\underset{\sim}{Y})|\underset{\sim}{X}) + \mathrm{var}\,\mathbb{E}(g(\underset{\sim}{X},\underset{\sim}{Y})|\underset{\sim}{X})$

KOVARIANSSIMATRIISI: · $\mathrm{Cov}(\underset{\sim}{X}) = \mathrm{cov}(\underset{\sim}{X},\underset{\sim}{X})$, $\mathrm{cov}(\underset{\sim}{X},\underset{\sim}{Y}) = \mathbb{E}(\underset{\sim}{X}-\mathbb{E}\underset{\sim}{X})(\underset{\sim}{Y}-\mathbb{E}\underset{\sim}{Y})^T$

· $\mathrm{cov}(A\underset{\sim}{X}, B\underset{\sim}{Y}) = A\,\mathrm{cov}(\underset{\sim}{X},\underset{\sim}{Y})B^T$ ← ja em.

$N_n(0, I_n)$: $\underset{\sim}{U} \sim N_n(0, I_n) \Leftrightarrow$ TF $f_{\underset{\sim}{U}}(\underset{\sim}{u}) = (2\pi)^{-n/2}\exp(-\frac{1}{2}\underset{\sim}{u}^T\underset{\sim}{u})$, $\mathbb{E}\underset{\sim}{U} = 0$, $\mathrm{Cov}(\underset{\sim}{U}) = I_n$.

$N_n(\mu, \Sigma)$: $\underset{\sim}{X} \overset{d}{=} A\underset{\sim}{U} + \mu$, $\mathbb{E}\underset{\sim}{X} = \mu$, $\mathrm{Cov}\,\underset{\sim}{X} = AA^T = \Sigma$. jos Σ kää.

$M_{\underset{\sim}{X}}(\underset{\sim}{t}) = \exp(\underset{\sim}{t}^T\mu + \frac{1}{2}\underset{\sim}{t}^T\Sigma\underset{\sim}{t})$, $f_{\underset{\sim}{X}}(\underset{\sim}{x}) = (2\pi)^{-n/2}(\det\Sigma)^{-1/2}\exp(-\frac{1}{2}(\underset{\sim}{x}-\mu)^T\Sigma^{-1}(\underset{\sim}{x}-\mu))$

TIHEYSFUNKT. MUUNTOKAAVA (NUOLISÄÄNTÖ)

$f_{\underset{\sim}{X}}(\underset{\sim}{x})|\partial\underset{\sim}{x}| = f_{\underset{\sim}{Y}}(\underset{\sim}{y})|\partial\underset{\sim}{y}|$, $\underset{\sim}{y} = g(\underset{\sim}{x}) \Leftrightarrow \underset{\sim}{x} = h(\underset{\sim}{y})$,

$$\frac{\partial\underset{\sim}{x}}{\partial\underset{\sim}{y}} = J_h(\underset{\sim}{y}) = \det(D_j h_i(\underset{\sim}{y}))$$