$$\mathbb{E}Y^2 = \int_{-\infty}^{\infty} y^2 f_Y(y)\,dy = 5\int_0^1 y^6\,dy = \frac{5}{7}.$$

$$\Rightarrow \operatorname{Var} Y = \frac{5}{7} - \underbrace{(\mathbb{E}Y)^2}_{=\frac{5}{6}\cdot\frac{5}{2\cdot 3}} = \frac{5}{7} - \frac{5^2}{2^2\cdot 3^2} = \frac{5}{2^2\cdot 3^2\cdot 7}\left(\underbrace{2^2\cdot 3^2}_{\substack{=4\cdot 9\\=36}} - \underbrace{5\cdot 7}_{=35}\right) = \frac{5}{4\cdot 9\cdot 7}$$

$$\mathbb{E}(XY) = \iint_{\mathbb{R}^2} xy\, f_{X,Y}(x,y)\,dx\,dy$$

$$= 15\int_0^1 dy \int_0^y dx\, \underbrace{xy\cdot x^2 y}_{=x^3y^2} = 15\int_0^1 dy\, \left(y^2 \underbrace{\Big/_0^y \tfrac{1}{4}x^4}_{=\frac{1}{4}y^4}\right)$$

$$= \frac{15}{4}\int_0^1 y^6\,dy = \frac{15}{7\cdot 4}$$

$$\mathbb{E}X\,\mathbb{E}Y = \frac{5}{8}\cdot\frac{5}{6} = \frac{5^2}{2^4\cdot 3}$$

$$\Rightarrow \operatorname{cov}(X,Y) = \mathbb{E}(XY) - \mathbb{E}X\,\mathbb{E}Y$$

$$= \frac{5\cdot 3}{2^2\cdot 7} - \frac{5^2}{2^4\cdot 3} = \frac{5}{2^4\cdot 3\cdot 7}\left(\underbrace{2^2\cdot 3^2}_{\substack{=4\cdot 9\\=36}} - \underbrace{7\cdot 5}_{=35}\right)$$

$$(*)\ \operatorname{var} X = \frac{2^6\cdot 3 - 7\cdot 5^2}{2^6\cdot 7} = \frac{17}{2^6\cdot 7} \qquad = \frac{5}{2^4\cdot 3\cdot 7}$$

$$\therefore \operatorname{Cov}(Z) = \begin{pmatrix} \frac{17}{2^6\cdot 7} & \frac{5}{2^4\cdot 3\cdot 7} \\ \frac{5}{2^4\cdot 3\cdot 7} & \frac{5}{2^2\cdot 3^2\cdot 7} \end{pmatrix}.$$

6) $\cdot\ f_Z(x,y) \overset{\text{cd. tabl}}{=} 15x^2y\, 1\!\!1\{0<x<y<1\}$

$\cdot\ f_X(x) = \frac{15}{2}x^2(1-x^2)\, 1\!\!1\{0<x<1\}$

$\cdot\ f_Y(y) = 5y^4\, 1\!\!1\{0<y<1\}$

a)

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}\, 1\!\!1\{f_X(x)>0\}$$

Ny * $\quad f_X(x) > 0 \iff 0 < x < 1$, jelen

$\mathbb{1}\{f_X(x) > 0\} = \mathbb{1}\{0 < x < 1\}$.

$$\Rightarrow \quad f_{Y|X}(y|x) = \frac{\cancel{15}\,\cancel{x^2}\,y}{\frac{\cancel{15}}{2}\cancel{x^2}(1-x^2)} \cdot \frac{\mathbb{1}\{0<x<y<1\}}{\mathbb{1}\{0<x<1\}} \cdot \mathbb{1}\{0<x<1\}$$

$$= \quad -\,\|\,- \quad \mathbb{1}\{x<y<1\} \cdot \mathbb{1}\{0<x<1\}$$

$$= \begin{cases} \dfrac{2y}{1-x^2}, & 0<x<y<1 \\ 0, & \text{máshol.} \end{cases}$$

$$= \begin{cases} \dfrac{2y}{1-x^2}\,\mathbb{1}\{x<y<1\} & \text{ha } 0<x<1 \\ 0 & \text{máshol} \end{cases}$$

b) $$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}\,\mathbb{1}\{f_Y(y)>0\}$$

Ny * $\quad f_Y(y) > 0 \iff 0 < y < 1$

jelen

$$f_{X|Y}(x|y) = \frac{\cancel{15}^{\,3}\,x^2\,\cancel{y}}{\cancel{5}\,y^{4}\,3} \cdot \frac{\mathbb{1}\{0<x<y<1\}}{\mathbb{1}\{0<y<1\}} \cdot \mathbb{1}\{0<y<1\}$$

$$= \begin{cases} \dfrac{3x^2}{y^3}\,\mathbb{1}\{0<x<y\} & \text{ha } 0<y<1 \\ 0 & \text{máshol.} \end{cases}$$

7.)

a) $\mathbb{E}(X \mid Y = y) = \int_{-\infty}^{\infty} x\, f_{X|Y}(x|y)\, dx$

ed. feld

másik

$$= \mathbb{1}\{0<y<1\} \int_{-\infty}^{\infty} x \cdot \frac{3x^2}{y^3}\, \mathbb{1}\{0<x<y\}\, dx$$

$$= \frac{\mathbb{1}\{0<y<1\}}{y^3} \int_0^y 3x^3\, dx = \frac{\mathbb{1}\{0<y<1\}}{y^3} \Big/_{0}^{y} \frac{3}{4} x^4$$

$$= \frac{3}{4} \frac{y^4}{y^3}\, \mathbb{1}\{0<y<1\} = \frac{3}{4} y\, \mathbb{1}\{0<y<1\}.$$

b) $\mathbb{E}(Y \mid X = x) = \int_{-\infty}^{\infty} y\, f_{Y|X}(y|x)\, dy$

$$= \frac{\mathbb{1}\{0<x<1\}}{1-x^2} \cdot \cancel{2y} \int_{-\infty}^{\infty} y \cdot 2y\, \mathbb{1}\{x<y<1\}\, dy$$

$$= \frac{\mathbb{1}\{0<x<1\}}{1-x^2} \int_x^1 2y^2\, dy = \frac{\mathbb{1}\{0<x<1\}}{1-x^2} \underbrace{\Big/_{x}^{1} \frac{2}{3} y^3}_{= \frac{2}{3}(1-x^3)}$$

$$= \frac{\mathbb{1}\{0<x<1\}}{\underbrace{1-x^2}_{=(1-x)(1+x)}} \cdot \frac{2}{3} \underbrace{(1-x^3)}_{=(1-x)(1+x+x^2)}$$

$$= \mathbb{1}\{0<x<1\} \cdot \frac{2}{3} \frac{1+x+x^2}{1+x}$$