

$$15) \int \begin{cases} X \sim U(2, 4) \\ Y|X \sim U(1, 3X) \end{cases}$$

a) Laske EY .

b) -- Var Y .

c) Laske yd. ff $f_{X,Y}$

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$$\begin{aligned} a) \quad EY &= E[E(Y|X)] = E\left(\frac{3X+1}{2}\right) = \frac{3}{2} EY + \frac{1}{2} \\ &= \frac{9}{2} + \frac{1}{2} = \frac{10}{2} = 5. \end{aligned}$$

$$\begin{aligned} b) \quad \text{Var } Y &= E \text{Var}(Y|X) + \text{Var } E(Y|X) \\ &= E\left(\frac{3X-1}{2}\right)^2 + \text{Var}\left(\frac{1}{2}(1+3X)\right) \\ &= \frac{1}{2} (9 EY^2 - 6 EY + 1) + \frac{1}{4} \text{Var}(3X) \\ &= \frac{3}{4} (\underbrace{\text{Var } X}_{= \frac{1}{12}(4-2)^2 = 2^2 = 4} + \underbrace{(EY)^2}_{= 3^2 = 9}) - \frac{1}{2} \cdot \underbrace{EY}_{= 3} + \frac{1}{2} + \frac{9}{4} \underbrace{\text{Var } X}_{= \frac{1}{3}} \\ &= \frac{1}{4} + \frac{3}{4} \cdot 9 - \frac{3}{2} + \frac{1}{2} + \frac{3}{4} = 1 + \frac{27}{4} - \frac{3}{2} + \frac{1}{2} \\ &= \frac{25}{4} + \frac{1}{2} = \frac{25+1}{2} = \frac{26}{2} = \frac{13}{1} = 13. \end{aligned}$$

$$\begin{aligned} c) \quad f_{X,Y}(x,y) &= f_X(x) f_{Y|X}(y|x) = \frac{1}{2} \mathbb{1}\{2 < x < 4\} \cdot \frac{1}{3x-1} \mathbb{1}\{1 < y < 3x\} \\ &= \frac{1}{6x-2} \mathbb{1}\{2 < x < 4, 1 < y < 3x\} \end{aligned}$$

huolellasti
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