1. Let $\alpha > 0$. Compute the dilatations $\mu_L = \partial L / \partial L$ and $\mu_R = \partial R / \partial R$ where 

$$L(x + iy) = (\alpha + 1)x + iy$$

is the linear stretch mapping and 

$$R(z) = z|z|^\alpha$$

is radial stretch mapping. Verify explicitly that $\mu_L = (e^z)^* \mu_R$.

2. Write down a formula for the Jacobian of a map $f : \mathbb{C} \to \mathbb{C}$ in terms of the complex derivatives $\partial f$ and $\bar{\partial} f$.

3. Construct an example of a map $f : E \to \mathbb{R}^2$, with $E \subset \mathbb{R}^2$, that is weakly quasisymmetric but not quasisymmetric.

4. Suppose $u \in C^2(\Omega)$ where $\Omega = \{ z : r < |z| < 1 \}$, with boundary values $u(re^{i\theta}) = 0$, $u(e^{i\theta}) = 1$, $0 \leq \theta \leq 2\pi$. Find the optimal lower bound for the energy 

$$E(u) = \int_{\Omega} |\nabla u|^2 dxdy.$$ 

5. Suppose $u : \Omega' \to \mathbb{R}$ is a Lipschitz function. Given $f \in W^{1,p}_{loc}$ with $f(\Omega) \subseteq \Omega'$, show that $u \circ f \in W^{1,p}_{loc}$.

A word on notation. Above, 

$$\partial f := \frac{\partial f}{\partial z}, \quad \bar{\partial} f := \frac{\partial f}{\partial \bar{z}}.$$ 

If $f$ is holomorphic, then 

$$f^* \left( \mu(z) \frac{dz}{dz} \right) = \mu(f(z)) \frac{f'(z)}{|f'(z)|} \cdot \frac{d\bar{z}}{dz}.$$