1. For $\alpha, \beta > 0$, let $f : \mathbb{R} \to \mathbb{R}$ that $f(x) = x^\alpha$ if $x > 0$ and $f(x) = -|x|^\beta$ for $x < 0$. Show that $f$ is quasisymmetric if and only if $\alpha = \beta$.

(Note that $f$ and $f^{-1}$ are Hölder continuous mappings for any $\alpha, \beta > 0$.)

2. (a) Show that the snowflake $K$ may be represented as the image of the line segment $[0, 1]$ under a $Ct^\alpha$-quasisymmetric map $f : [0, 1] \to K$ for some $\alpha < 1$.

(b) Nevertheless, show that one cannot take $\alpha$ to be arbitrarily close to 1.

3. Suppose $f : \mathbb{D} \to \mathbb{C}$ is a conformal mapping that admits an $\eta$-quasisymmetric extension to the plane. For an arc $I \subset S^1$, define its conformal midpoint $z_I$ as the midpoint of the hyperbolic geodesic joining $z_1$ and $z_2$. Show that there exists a constant $C$ (depending on $\eta$) so that

$$\frac{1}{C} |f'(z_I)| \leq \frac{\text{diam } f(I)}{\text{diam } I} \leq C |f'(z_I)|$$

(Hint: use Koebe’s distortion theorem.)

4. Let

$$f(z) = z + \frac{b_1}{z} + \frac{b_2}{z^2} + \frac{b_3}{z^3} + \ldots$$

be a conformal map of the exterior unit disk $\{z : |z| > 1\}$ to a domain $\Omega = \mathbb{C} \setminus K$. The aim of this problem is to find a formula for the area of $K$ in terms of the coefficients $b_k$. 
(a) Find an asymptotic formula (as \( R \to \infty \)) for the area \( A(R) \) of the compact set enclosed by the curve \( f(S_R) \), where \( S_R = \{ z : |z| = R \} \).

(b) Compute

\[
B(R) = \lim_{\rho \to 1} \int_{\rho <|z|<R} |f'(z)|^2.
\]

by using the power series expansion.

(c) Analyze \( \lim_{R \to \infty} (A(R) - B(R)) \).