A The ZFC Axioms

The theory is formulated in first-order logic with identity and the binary relation symbol $\in$.

1. *Set existence*: $\exists x(x = x)$.

2. *Extensionality*: $\forall x \forall y(\forall z(z \in x \leftrightarrow z \in y) \to x = y)$.

3. *Foundation*: $\forall x\,(\exists y(y \in x) \to \exists y(y \in x \wedge \neg \exists z(z \in x \wedge z \in y)))$.

4. *Comprehension Scheme*: For each formula φ with free variables among $x, z, w_1, \ldots, w_n$, $\forall z \forall w_1, \ldots \forall w_n \exists y \forall x(x \in y \leftrightarrow x \in z \wedge \varphi)$.

5. *Pairs*: $\forall x \forall y \exists z(x \in z \wedge y \in z)$.

6. *Unions*: $\forall z \exists x \forall y \forall w(w \in y \wedge y \in z \to w \in x$.

7. *Replacement Scheme*: For each formula φ with free variables among $x, y, z, w_1, \ldots, w_n$, $\forall z \forall w_1 \ldots \forall w_n\,(\forall x \in z \exists! y \varphi \to \exists v \forall x \in z \exists y \in v \varphi)$.

8. *Infinity*: $\exists x(0 \in x \wedge \forall y \in x(S(y) \in x))$. *Power set*: $\forall x \exists y \forall z(z \subset x \to z \in y)$.

9. *Choice*: $\forall x \exists y('y$ is a function with domain $x' \wedge \forall z(z \in x \wedge \exists w(w \in z) \to y(z) \in z))$.

In axioms 7-9 one uses the fact that the earlier axioms allow one to define $\subset$ (subset), 0 (empty set), S (ordinal successor), the property $\exists! y$ 'there exists a unique y' and the property 'f is a function with domain y' as well as the element $f(z)$ for a function f with z in its domain.

For a thorough treatment of the axioms of ZFC, see e.g. [End77],[Kun80] or [Jec03].