Classification theory

Exercise 1

1. Suppose $\pi : \mathcal{A} \to \mathcal{B}$ is a bijection. Show (straight from the definitions) that the following are equivalent:
(i) For all atomic formulas $\phi(x)$ and $a \in \mathcal{A}$, $\mathcal{A} \models \phi(a)$ iff $\mathcal{B} \models \phi(\pi(a))$.
(ii) (a)-(c) hold, where
(a) for all relation symbols R and $a \in \mathcal{A}$, $a \in R^{\mathcal{A}}$ iff $\pi(a) \in R^{\mathcal{B}}$,
(b) for all function symbols f and $a \in \mathcal{A}$, $\pi(f^{\mathcal{A}}(a)) = f^{\mathcal{B}}(\pi(a))$,
(c) for all constant symbols c, $\pi(c^{\mathcal{A}}) = c^{\mathcal{B}}$.

2. Suppose $\mathcal{A}$ is a model, B is a set and π is a bijection from the universe of $\mathcal{A}$ onto B. Show that there is a model $\mathcal{B}$ such that its universe is B and π is an isomorphism

3. Suppose $A \subseteq \mathcal{A}$, $\mathcal{A}$ is strongly κ-homogeneous, $|A| < \kappa$ and $a, b \in \mathcal{A}$. Show that if $t(a/A; \mathcal{A}) = t(b/A; \mathcal{A})$, then there is an automorphism f of $\mathcal{A}$ such that $f \restriction A$ is identity (i.e. $f \in Aut(\mathcal{A}/A)$) and $f(a) = b$.

4. Show that the theory T_ω (from the introduction of the lecture notes) has elimination of quantifiers.

5. Exercise 6.7 from the lecture notes of the course Model theory (these can be found from the home page of Introduction to classification theory).