Topology II | Homework set 0
Spring 2015 | (ex tempore)
Tue 13.1.2015

Exercise 1

Let X be a set. Suppose that J is an indexing set for a family A_j, $j \in J$, of subsets of X. Show that then for every $B \subset X$

$$B \cap \Big(\bigcup_{j \in J} A_j \Big) = \bigcup_{j \in J} (B \cap A_j) \ .$$

Exercise 2

Consider functions $f : X \to Y$, $g : Y \to Z$ between the sets X, Y, Z. Prove the following statements:

(a) $(g \circ f)[A] = g[f[A]]$ for all $A \subset X$.
(b) $(g \circ f)^{\leftarrow}[C] = f^{\leftarrow}[g^{\leftarrow}[C]]$ for all $C \subset Z$.

(**Notations:** Here $g \circ f : X \to Z$ denotes the composite function, $f[A] := \{f(x) \,|\, x \in A\}$ denotes an image, and $g^{\leftarrow}[C] := \{y \in Y \,|\, g(y) \in C\}$ a preimage. In the course textbook and the lectures, the last two are typically denoted by "fA" and "$g^{\leftarrow}C$" or "$g^{-1}C$".)

Exercise 3

Minitopology

Consider a set X and define $\mathcal{T}_{\text{mini}} := \{\emptyset, X\}$.

(a) Show that $\mathcal{T}_{\text{mini}}$ is a topology on X.
(b) Is always $\#\mathcal{T}_{\text{mini}} = 2$? (In other words, does the set always have two elements?)
(c) Suppose that $\#X \geq 2$ and that $d : X \times X \to \mathbb{R}_+$ is a metric on X. Find an open ball in the metric d which does *not* belong to $\mathcal{T}_{\text{mini}}$. (This shows that, if a set has at least two elements, its minitopology is not metrizable.)

Exercise 4

Consider a function $f : X \to Y$ between sets X and Y. Show that then for all $B \subset Y$

$$f^{\leftarrow}[\complement B] = \complement f^{\leftarrow}[B] \,.$$