Inverse Problems Project: Pistachio

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Numerical implementation with Barzilai-Borwein method

Minimizing the objective function \( \| \) is a numerical optimization problem. For problems where the objective function has a known, continuous gradient that can be efficiently computed, many last gradient-based optimization algorithms are available, one of them the Barzilai-Borwein (B-B) method. However, \( \| \) isn’t continuously differentiable, so to use Barzilai-Borwein, we approximate the absolute value function \( | \) in \( \| \) by \( | | ) , where \( \beta > 0 \) small, which naturally has a continuous derivative, and so in numerical calculations we replace \( \| \) with an approximation \( \| \) that can now be analytically solved (see [5] Eq. 1.6.3 – 1.6.4).

\[
\| f \|_{TV} = \beta \| \nabla f \|_{2}.
\]

The B-B method (presented in [1]) is a version of the famous iterative gradient descent or steepest descent algorithm, and it can be described as follows:

After some initial guess \( f^{(0)} \), each iteration step \( f^{(n)} \) is determined from the previous step \( f^{(n-1)} \) by

\[
f^{(n+1)} = f^{(n)} - \delta_{n} \| \nabla g(f^{(n)}) \|_{2}
\]

where the steplength parameter \( \delta_{n} \) is given by

\[
\delta_{n} = \frac{\| g_{n} \|_{2}}{\| \nabla g_{n} \|_{2}}
\]

and \( g_{n} = f^{(n)} - f^{(n-1)} \) and \( g = \| \nabla g(f^{(n)}) - \| \nabla g(f^{(n-1)}) \|_{2} \) .

Automatic regularization parameter choice

In any implementation of total variation regularization, an important question is how to choose the regularization parameter \( \alpha \) . In this project, we tested an automatic TV norm based method inspired by the sparsity S-curve method proposed in [6] Section 6.3 and [7] 3.

Our ‘measurement of sparsity’ is simply the TV norm of the (reconstructed) slice image: we noticed that reconstructions with very small \( \alpha \) have quite much ‘variation’ measured by \( \| f \|_{TV} \) , and likewise with very large \( \alpha \) , \( \| f \|_{TV} \) is small; moreover, curve seems to decrease monotonically (at least in a feasible range). Assuming we have a priori knowledge of the desired level of total variation in a good reconstruction, we can find a good guess for \( \alpha \) with an interpolation method similar to the ‘S-curve’.

We compute the reconstructions and their TV norms for multiple but computationally feasible number of discrete points \( \alpha_{1}, \ldots, \alpha_{k} \) in some range. Our automatic guess is the point \( \alpha_{k} \) that has an interpolated (piecewise cubic interpolation) value nearest to the a priori known TV value. (See Figure 4)

To create a priori data similar enough to sparse B-B reconstructions, we used full-angle reconstruction obtained by simple filtered back-projection (FBP) with some noise reduction in addition basic X-ray image preprocessing: Reconstruction image was first filtered using MATLAB’s wiener2.m routine, and then still noisy background was averaged. See Figure 5 for resulting a priori data.

The FBP couldn’t itself be used because full angle FBP (Figure 2) reconstructed more imaging noise (thus variation) than the sparse reconstructions with 15 – 30 angles (e.g. 6).

DISCUSSION

Reconstructions with both 30 and 15 angles turned out surprisingly good, 30 slightly but noticeably better. TV managed to reconstruct even the small details clear and sharp, but in the 15 angles version some artifacts were shown.

Even though results are not fully reliable since our a priori data was computed from the same full-angle data set than actual reconstructions, this study gives a good demonstration of TV as a tomography reconstruction method and points out TV’s power to create sharp, well-detailed reconstructions.

REFERENCES


