Affine and convex sets

Essential concepts: affine set, convex set, affine hull, convex hull, affine mapping, affine dimension of a convex set, affine combination of vectors, simplicial combination of vectors.

Every affine set is a translation of the unique vector subspace.
Lemma 2.7. - characterization of affine and convex hull in terms of affine/convex combinations.

Simplices

Essential concepts: affinely independent set (Lemma 2.10 gives important equivalent ways to define), simplex (a convex hull of affinely independent set), vertices of simplex, dimension of simplex, faces of simplex, simplicial interior, simplicial boundary, ordered simplex, the index of a face $d_i\sigma$ in the ordered simplex σ. Standard n-simplex Δ_n.

Lemma 2.15 - in order to construct an affine mapping on the simplex, it is enough to tell where vertices go.

Topology on simplices

Every simplex has a natural standard topology. All simplices of the same dimension are homeomorphic. Simplex is path-connected, even contractible and compact. The topological interior and boundary coincide with simplicial **if** calculated with respect to the affine hull of the simplex.

Lemma 3.19 and Theorem 3.20 - important topological properties of closed bounded convex subsets of $\mathbb{R}^n$.

Simplicial complexes

Simplicial complex is the collection of simplices satisfying conditions of Definition 4.1. Lemma 4.2. give an important equivalent way to define simpl. complex.

Polyhedron of a simpl. complex is a topological space associated with the complex. The topology of the polyhedron is **weak topology** (important concept!). It is coherent with the standard topologies of the simplices of complex. Simplices and their boundaries are always **closed** in polyhedron, but simplicial interiors USUALLY ARE NOT!

Polyhedrons of finite complexes are compact.

Important concepts: vertices of complex, subcomplexes (and its properties), nth sceleton, dimension of a complex, triangulation of top. spaces. , star of an element.

Subdivision

In practise it is enough to be familiar with so-called **barycentric subdivision** and its properties. The essential properties are summarized in Lemma 4.17, Lemma 4.18 and Corollary 4.19. These properties make it possible to prove one of the main results in the theory of simplicial complexes - proposition 4.21 and especially Corollary 4.22.

Simplicial mappings and simplicial approximation

Essential concepts: simplicial mapping, simplicial approximation of a continuous mapping between polyhedra.

Corollary 5.2. gives a convenient way to check whether the mapping between polyhedra is continuous.

Lemma 5.8. - simplicial approximation is homotopic to original mapping.

Lemma 5.9 - convenient tecnical result.

Theorem 5.10 - simplicial approximatin theorem, one of the main results. Its Corollary 5.11 also important.

Δ-complex

Enough to grasp the idea - spaces are build out of simplices by identifying some faces of some simplices together. Important to understand and know the formalization of gluing i.e. quotient maps and quotient spaces and their properties. Proposition 6.5. is the one that is used in practise when showing that space Y is homeomorphic to a quotient space of some other space X. It is important to know how this result is applied in practise.

Triangulating spaces as polyhedrons of Δ-complexes - enough to be able to describe simplices, order them and describe all the essential identifications of simplices. Adobt " model " from Examples 6.15 - 6.17.

Example 6.18 illustrates "cut and glue" technique.

Abelian groups

This is all supposed to be known from earlier algebra courses. Most important skill is too master quotient groups and how to apply Isomorphism theorem 7.9. (or more general Factorization Theorem 7.8) in practise.

Free Abelian groups

Essential concepts: integers as "scalars" in abelian group, linear combination in abelian group, free subset, basis of an abelian group, free group, finitely supported families, formal sums over finitely supported families.

Lemma 8.4. - To construct homomorphism from a free abelian group enough to choose how basis elements are mapped. This choice is completely free of conditions.

Groups $\mathbb{Z}^A$ and $\mathbb{Z}^{(A)}$. The latter turns out to be universal " model " for free abelian group (Corollary 8.10).

Direct product and direct sum. The essential universal property of direct sum is given in Lemma 8.13. Also the property mentioned in 8.16 is equally important to know.