Answers / Ex 4

4.1. The estimated standard deviation is s = 0.05 (seconds). Determine n such that the estimation error for the mean reaction time is less than 0.01.
a) with 95 % confidence. Start by using normal distribution and z-value = 1.96.
The estimation error is

$$\pm\ 1.96\ s / \sqrt{n} = 1.96 \times 0.05\ \sqrt{n}\ (= \Delta)$$

Want: $\Delta \leq 0.01$. n can be solved from the equation (you may use an inequality as well):

$$0.01 = 1.96 \times 0.05 / \sqrt{n} \;=>\; \sqrt{n} = 1.96 \times 0.05 / 0.01 \;=>\; \sqrt{n} = 9.8$$

Thus: $n = (9.8)^2$, leading to $n \geq 97$.
REMARK: Since n > 30, the use of the normal distribution is justified.
b) with 99 % confidence. Repeat he previous calculations by using the z-value = 2.58.
Thus, n can be solved from the equation (you may use an inequality as well):

$$0.01 = 2.58 \times 0.05 / \sqrt{n} \;=>\; \sqrt{n} = 2.58 \times 0.05 / 0.01 \;=>\; \sqrt{n} = 12.9.$$

Thus: $n = (9.8)^2$, leading to $n \geq 167$.

4.2. The estimated standard deviation is s = 100 (hours). Determine n such that the estimation error for the mean life time of the lamps is less than a) 20 hours, b) 10 hours.
a) Repeat the argumentation used in 4.2. with $\Delta = 20$, yielding

$$20 = 1.96 \times 100 / \sqrt{n} \;=>\; \sqrt{n} = 1.96 \times 100 / 20 \;=>\; \sqrt{n} = 1.96 \times 5 = 9.8$$

Thus: $n = (9.8)^2$, leading to $n \geq 97$.
b) Now $\Delta = 10$. Thus:

$$10 = 1.96 \times 100 / \sqrt{n} \;=>\; \sqrt{n} = 1.96 \times 100 / 10 \;=>\; \sqrt{n} = 1.96 \times 10 = 19.6$$

Thus: $n = (19.6)^2$, leading to $n \geq 385$

4.3. 40 tosses of a coin <=> n = 40. Result 24 heads. Determine the confidence limits for p = P(Head) by using the estimate p* = 24/40 = 0.6. (p* is used her ONLY for typographical convenience)
a) 95 % confidence limits: 40 > 30 => we may use the z-value 1.96
The confidence limits are:

$$p^* \pm 1.96 \sqrt{p^*(1-p^*) / n} \;<=>\; 0.6 \pm 1.96 \sqrt{0.6 \times 0.4 / 40} \;<=>\; 0.6 \pm 0.15$$

Thus, the 95 % confidence interval is: $0.45 < p < 0.75$.
b) 99 % confidence. Repeat the calculations above by using the z-value = 2.58.
We get $0.6 \pm 2.58 \sqrt{0.6 \times 0.4 / 40} \;<=>\; 0.6 \pm 0.23$
Thus, the 99 % confidence interval is: $0.37 < p < 0.83$.

4.4. 100 voters => n = 100 and p* = 0.55 (= 55 %)
a) 95 % confidence limits for p can be obtained by repeating the calculation in 4.4. with n = 100 and p* = 0.55 yielding

$$0.55 \pm 1.96 \sqrt{0.55 \times 0.44 / 100} \;<=>\; 0.55 \pm 0.10$$

Thus, the 95 % confidence interval is: $0.45 < p < 0.65$.
b) 99 % confidence. Repeat the calculations above by using the z-value = 2.58.
We get $0.55 \pm 2.58 \sqrt{0.55 \times 0.45 / 100} \;<=>\; 0.55 \pm 0.13$
Thus, the 99 % confidence interval is: $0.42 < p < 0.68$.

4.5. Observed mean = 1570, s = 120.
$H_o : \mu = 1600$, $H_1 : \mu \neq 1600$ (n = 100).
Two-tailed test. Significance level $\alpha = 0.05$. n = 100 => may use normal distr.
=> Critical values $\pm$ 1.96
Under H_o we have: $z = (m - 1600) / (120 \text{ x}10) = (1570 - 1600) /12 = - 2.50$
(within the critical region). Conclusion: We reject H_o at a 0.05 significance level.

$\alpha = 0.05$ Critical values $\pm$ 2.56. Now the observed value = - 2.50 is not within the critical region. => We cannot reject H_o at 0.01 significance level.

4.6. Suppose two classes come from two populations having the respective means μ_1 and μ_2. Test

$$H(0): \mu_1 = \mu_2 \text{ against } H(1): \mu_1 \neq \mu_2 \text{ (two-tailed test)}$$

Observed $m_1 = 74$ ($n_1 = 40$) and $s_1 = 8$
$m_2 = 78$ ($n_2 = 50$) and $s_2 = 7$

H(0) true: The estimated variance of the test statistic $m_1 - m_2$ is

$$s = \sqrt{(8^2/40 + 7^2 / 50)} = 1.606$$

=> the test statistic is

$$z = (74 - 78) / 1.606 = - 2.49$$

a) $\alpha = 0.05$ Critical values of z are $\pm$ 1.96. Thus, there is a statistically significant difference in the performance of the two classes at the level 0.05

b) $\alpha = 0.01$ No statistically significant difference at the level 0.01.