Malliteoria

Harjoitus 3

1. Exercise 3.8.

2. Exercise 3.9.

3. Suppose $A \subseteq \mathcal{A}$ and $B = \{t^{\mathcal{A}}(a) |\ t(x_1, ..., x_n)$ is a term and $a \in \mathcal{A}^n\}$. Show that for all $f \in L$ and $b \in B^{\#f}$, $f^{\mathcal{A}}(b) \in B$. Conclude that $\mathcal{A} \restriction B$ is the submodel generated by A, see Exercise 4.2 (v).

4. Suppose $f : \mathcal{B} \to \mathcal{A}$ is a partial isomorphism, $\mathcal{C}$ is the submodel generated by $dom(f)$ and define $g : \mathcal{C} \to \mathcal{A}$ so that for all terms $t(x_1, ..., x_n)$ and $a \in dom(f)^n$, $g(t^{\mathcal{B}}(a)) = t^{\mathcal{A}}(f(a))$. Show that g is well-defined and an embedding.

5. Exercise 4.2 (vi).

6. Assume $\mathcal{A}$ is a structure.

(i) Suppose $\phi(v_0, x)$, $x = (x_0, ..., x_n)$, is a formula, $a \in \mathcal{A}^n$ and $\phi(\mathcal{A}, a)$ is infinite. Show that there is $\mathcal{B} \succeq \mathcal{A}$ such that $\phi(\mathcal{B}, a) \not\subseteq \mathcal{A}$.

(ii) Suppose $X \subseteq \mathcal{A}$ is infinite. Show that there is $\mathcal{B} \succeq \mathcal{A}$ such that X is not definable in $\mathcal{B}$. Hint: Using (i), build a suitable elementary chain.