Inverse problems course, spring 2014 Exercise 1 (January 21-24, 2014)
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Theoretical exercises:

T1. Define a function $g : \mathbb{R} \to \mathbb{R}$ by

$$g(x) = \begin{cases} 1 & \text{for } 0.4 \le x \le 0.6, \\ 0 & \text{otherwise.} \end{cases}$$

Compute the function $g * g$ analytically (by hand), where

$$(g * g)(x) = \int_{-\infty}^{\infty} g(x')g(x - x')\, dx'.$$

Outside which interval $[a, b] \subset \mathbb{R}$ is $(g * g)(x) = 0$?

T2. Let the discrete point spread function $p \in \mathbb{R}^5$ and the vector $f \in \mathbb{R}^{10}$ be defined by

$$\begin{aligned} \widetilde{p} &= [\widetilde{p}_{-2}, \widetilde{p}_{-1}, \widetilde{p}_0, \widetilde{p}_1, \widetilde{p}_2]^T = [1, 1, 1, 1, 1]^T, \\ f &= [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}]^T = [0, 0, 0, 0, 1, 1, 0, 0, 0, 0]^T. \end{aligned}$$

Compute the discrete convolution vector $(\widetilde{p} * f) \in \mathbb{R}^{10}$ by

$$(\widetilde{p} * f)_j = \sum_{\ell=-2}^{2} \widetilde{p}_\ell f_{j-\ell},$$

where $f_{j-\ell}$ is defined using periodic boundary conditions for the cases $j - \ell < 1$ and $j - \ell > n$.

T3. Take $\Delta x = \frac{1}{10}$ and compute the normalized point spread function

$$p = \left(\Delta x \sum_{j=-2}^{2} \widetilde{p}_j\right)^{-1} \widetilde{p}.$$

Compute the discrete convolution vector $(\widetilde{p} * f) \in \mathbb{R}^{10}$ with vector $f \in \mathbb{R}^{10}$ as in exercise T2 except that $f_1 = 2$. Be careful with the periodic boundary condition!

Matlab exercises:

M1. Download the following files from the course webpage:
`DC_PSF.m`
`DC_target.m`
`DC_convmtx.m`
`DC1_cont_data_comp.m`
`DC1_cont_data_plot.m`
`DC2_discretedata_comp.m`
`DC2_discretedata_plot.m`

(a) Repeat the experiment done at the lecture: choose `n = 32`, `n = 64`, `n = 128` and `n = 256` in line 12 of the file `DC2_discretedata_comp.m` and observe how the approximation error becomes smaller. (In other words, the blue dots in the image entitled *Data with inverse crime* get closer to the red function as `n` grows.)

(b) Now choose a to be smaller than 0.04 in line 10 of file `DC1_cont_data_comp.m` and run it. Repeat the experiment in (i). Is the convergence of blue dots to the red function slower or faster, especially near the discontinuities of the original signal? Why is this?

(c) Now choose a to be greater than 0.04 in line 10 of file `DC1_cont_data_comp.m` and run it. Repeat the experiment in (i). Is the convergence of blue dots to the red function slower or faster? Why?