1. Let $A$ be the infinitesimal generator of a strongly continuous semigroup $T$ with growth bound $\omega_0$, that is, $\omega_0 := \inf \{ \omega \in \mathbb{R} : \exists M \geq 1, \| T(t) \| \leq Me^{\omega t} \text{ for all } t \geq 0 \}$. Show that $\rho(A^*)$ contains the right half-plane $\{ \lambda \in \mathbb{C} : \text{Re} \lambda > \omega_0 \}$ and that $R(\lambda, A^*) = R(\lambda, A)^*$ for all $\lambda \in \mathbb{C}$ with $\text{Re} \lambda > \omega_0$.

2. Let $X$ be a Banach space and let $f : [a, b] \to X^*$ be continuous from $[a, b]$ to $X^*$ equipped with the weak*-topology. Show that the mapping

$$x \mapsto \int_a^b \langle x, f(s) \rangle ds$$

defines a continuous linear functional on $X$, that is, an element of $X^*$. This element is denoted $\int_a^b f(s) ds$ and is called the weak* Riemann integral of $f$. (Hint: Banach-Steinhaus).

3. Let $A$ be the infinitesimal generator of a strongly continuous semigroup $T$ on the Banach space $X$. Prove that $X$ is $\ominus$- reflexive with respect to $T$ if and only if $R(\lambda, A)$ is a compact operator in the weak topology on $X$ determined by $X^\ominus$, that is, the weakest topology on $X$ making the functionals $x \mapsto \langle x, x^\ominus \rangle$ continuous for all $x^\ominus \in X^\ominus$.

4. Let $A$ be the infinitesimal generator of a strongly continuous semigroup $T$ on the Banach space $X$. Prove that $X$ is $\ominus^\ominus$- reflexive with respect to $T$ if and only if $X^\ominus$ is $\ominus$- reflexive with respect to $T^\ominus$. 

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