

## Galactic dynamics – Problem set 6. Spring 2021

The answers should be returned by **Tuesday (4.5) 4pm (16.00) in Moodle**, link through the official course homepage. The answers to the problem set will be discussed on **Thursday (6.5) at 14.15-16.00 on Zoom**.

1. At typical sea-level conditions ( $p = 1.01 \times 10^5 \text{ N/m}^2$  and  $T = 15^\circ\text{C}$ ), the density of air is  $\rho = 1.22 \text{ kg/m}^3$  and the speed of sound is  $v_s = 340 \text{ m/s}$ . Find (i) the fractional change in frequency due to the self-gravity of the air, for a sound wave with wavelength  $\lambda = 1 \text{ m}$  and (ii) the Jeans length.
2. Prove that a system of  $N$  self-gravitating point masses with positive energy must disrupt, in the sense that at least one star must escape. Hint: use the virial theorem, and prove that the moment of inertia must increase without limit.
3. Consider a system in which the interparticle potential energy has the form  $\Phi_{\alpha\beta} = -C|\mathbf{x}_\alpha - \mathbf{x}_\beta|^{-p}$ , where  $p$  and  $C$  are positive constants.

(a) Show that the scalar virial theorem has the form

$$2K + pW = 0,$$

where  $K$  is the kinetic energy and  $W$  is the potential energy.

- (b) For what values of  $p$  does the system have negative heat capacity, in the sense of Equation 7.51 in the lecture notes.
4. Two identical galaxies are initially at rest, at a large distance from one another. They are spherical, composed solely of identical stars, and their light distributions obey the Sérsic law (eq 1.17 in the lecture notes) with Sérsic index  $m$  and effective radius  $R_e$ . The galaxies fall together and merge. If the merger product also satisfies the Sérsic law with the same index  $m$ , what is its effective radius?
  5. Show that the mass arm (the maximum of the surface density) in a tightly wound spiral leads the potential arm (the minimum of the gravitational potential) at a given radius by an angle

$$\Delta\phi = \frac{1}{km} \frac{d}{dR} \ln R^{1/2} |\Phi(R)|$$

where  $|\Phi(R)|$  is the amplitude of the spiral potential and the result has fractional error  $\mathcal{O}(|kR|^{-1})$ . Thus the mass arm can either lead or lag the potential arm, depending on the radial dependence of the strength of the spiral. Hint: use equation (6.33) in the lecture notes.