

Galactic dynamics – Problem set 4. Spring 2021

The answers should be returned by **Tuesday (23.3) 4pm (16.00) in Moodle**, link through the official course homepage. The answers to the problem set will be discussed on Thursday (25.3) at 14.15-16.00 on Zoom.

1. Prove that the density of a spherical, ergodic, self-consistent stellar system must decrease outward. Hint: in the integral for ρ make Φ the integration variable.
2. Show that in a frame that rotates with constant angular velocity $\boldsymbol{\Omega}$ with the effective potential $\Phi_{\text{eff}} = \Phi - \frac{1}{2}|\boldsymbol{\Omega} \times \mathbf{r}|^2$, the collisionless Boltzmann equation can be written as:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - [2(\boldsymbol{\Omega} \times \mathbf{v}) + \nabla \Phi_{\text{eff}}] \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

3. Show that a Hernquist model with constant anisotropy $\beta = \frac{1}{2}$ has

$$N(E) = \frac{3a}{GM} \tilde{\epsilon}^2 \left(\frac{1}{\tilde{\epsilon}} - 1 \right)^2$$

where $\tilde{\epsilon} = \epsilon a / (GM)$ and M and a are the mass and the scale radius of the Hernquist sphere.

4. A self-consistent stellar system has an ergodic DF and a power-law density profile $\rho = \rho_0(r_0/r)^\alpha$ with $1 < \alpha < 3$. Show that the velocity dispersion is given by

$$\overline{v_r^2}(r) = \frac{2\pi G \rho_0 r_0^\alpha r^{2-\alpha}}{(3-\alpha)(\alpha-1)} \quad (\alpha \neq 2).$$

What does this formula become in the case $\alpha = 2$ of the singular isothermal sphere?

5. Prove that the following DF generates a stellar system in which the density distribution is that of a homogeneous sphere of density ρ and radius a :

$$f(E, L) = \frac{9}{16\pi^4 G \rho a^5} \frac{1}{\sqrt{L^2/a^2 + \frac{4}{3}\pi G \rho a^2 - 2E}} \quad \left(L^2 < \frac{4}{3}\pi G \rho a^4 \right).$$

Here it is understood that $f = 0$ when the argument in the square root is not positive, the DF is normalised so that $\int d^3\vec{x} d^3\vec{v} f = 1$, the potential $\Phi = 0$ at $r = 0$, and the system is isolated.