Galaxy formation and evolution – Problem set 4. Autumn 2020

The answers should be returned by ***Wednesday (4.11) 4pm (16.00)*** *by email to the course assistant Stuart McAlpine (stuart.mcalpine@helsinki.fi). Please put "Galaxy formation – Problem set 4" in the title of your email. – The problem set will be discussed on Friday (6.11) after the lecture (at 14.15) on Zoom.*

1. Consider a population of galaxies with a comoving number density of $\bar{n} = 0.1h^3$ Mpc^{-3}

   (a) Let us assume that these galaxies are randomly distributed, in a Poisson process. In this case, calculate what is the probability that a random galaxy has no neighbours closer than $1h^{-1}$ Mpc.

   (b) What is the corresponding probability if the galaxies are clustered with a two-point correlation function:

   $$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

   with $r_0 = 2h^{-1}$ Mpc and $\gamma = 1.8$.

   Hint: Assume Poisson sampling. See paragraph 6.1.2 in MBW.

2. Spherical collapse model.

   (a) According to the spherical collapse model, the parametric solution to the evolution of a mass shell is given by:

   $$a = A(1 - \cos\theta) \qquad t = B(\theta - \sin\theta),$$

   where $A^3 = GMB^2$ and $\theta \in [0, 2\pi)$. Show that this implies that the overdensity δ can be expressed as:

   $$1 + \delta = \frac{9}{2}\frac{(\theta - \sin\theta)^2}{(1 - \cos\theta)^3}.$$

   (b) Show also that at early times (when $\theta \ll 1$) one has that:

   $$\delta_i = \frac{3}{20}(6\pi)^{2/3}\left(\frac{t_i}{t_{\rm max}}\right)^{2/3}$$

   Hint: Use the Taylor series expansions of $\sin\theta$ *and* $\cos\theta$ *and the fact that* $t_{\rm max} = \pi B$.

3. The epoch of galaxy formation.

   (a) The observed number density of bright galaxies today is $n_g = 10^{-3}$ Mpc^{-3}. Assuming that each galaxy has a total (dark matter + baryon) mass of $10^{12} M_\odot$. What fraction of the total mass of the Universe has collapsed into these galaxies? Assume $\Omega_{m,0} = 0.3$.

   (b) The distribution of the values of the overdensity $\delta = \Delta\rho/\rho$ at different locations is given by a Gaussian with an r.m.s. amplitude of $\sigma = \langle\delta^2\rangle^{1/2}$. Suppose each spherical region that has an overdensity of $\delta \geq \nu_g\sigma$ grows into a galaxy by the present epoch. Find the value of ν_g. *Hint: You will need erfc, the complementary error function.*

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4. Download and read the paper: *"Different Star Formation Laws for Disks Versus Starbursts at Low and High Redshifts"* by Daddi, Elbaz, Walter et al., 2010, ApJL, 714, L118 by using the link on the course homepage. Based on the paper answer the questions below:

   (a) What is the Schmidt-Kennicutt relation for star formation? What does it describe and how has it been derived?

   (b) Throughout the paper a conversion factor $\alpha_{\rm CO}$ is used. How is this factor defined and why is CO observations used instead of direct H_2 observations? Is $\alpha_{\rm CO}$ an universal constant or does it vary from galaxy to galaxy? If $\alpha_{\rm CO}$ varies, can you think of any reason why?

   (c) What does Figs. 1. and 2. describe in the paper? What is meant with the sequences for disks and starbursts and what types of objects are found on each sequence?

5. (a) What is meant by the gas consumption timescale and the dynamical timescale? How are these timescales defined? How is a starburst defined in the paper? Why are objects such as M82 and the nucleus of NGC 253 starbursts, although they have relatively modest star formation rates of only a few $M_{\odot}{\rm yr}^{-1}$?

   (b) Why is Fig 3. so different from Figs 1. and Figs 2.? Why in particular do all the objects now fit on a universal relation? What are the implications of Fig 3. for the star formation law in galaxies?

   (c) What does the right panel of Fig 4. show? What could be a possible explanation for the different ratio of the gas consumption and dynamical timescales for different objects? What is the role of the stellar initial mass function and galaxy mergers in explaining the observed results?