Galaxy formation and evolution – Problem set 3. Autumn 2020

The answers should be returned by ***Wednesday (14.10) 4pm (16.00)*** *by email to the course assistant Stuart McAlpine (stuart.mcalpine@helsinki.fi). Please put "Galaxy formation – Problem set 3" in the title of your email. – The problem set will be discussed on Friday (16.10) after the lecture (at 14.15) on Zoom.*

1. Perturbation analysis in the relativistic case. *Hint: Repeat the steps of the perturbation analysis of the non-relativistic case discussed during the lectures.*

   (a) Starting from the relativistic versions of the equation of continuity, the equation of motion and the Poisson equation presented in the lecture notes (Lecture 4, page 17) derive the following equation for the growth of density perturbations, Δ, in the relativistic plasma:

   $$\frac{d^2\Delta}{dt^2} + 2\left(\frac{\dot{a}}{a}\right)\frac{d\Delta}{dt} = \Delta\left(\frac{32\pi G\rho}{3} - k^2 c_s^2\right)$$

   (b) From this equation derive an expression for the relativistic Jeans' mass.

   (c) Finally, solve the perturbation equation for perturbations on scales much larger than the Jeans' length for which the pressure gradient term can be neglected and show that the growing solution corresponds to $\Delta \propto t \propto a^2 \propto (1+z)^{-2}$.

2. The Mészáros effect. *Hint: This problem is discussed in many textbooks, for example see the Longair book Section 12.6.* The growth of baryonic perturbations embedded in a uniform radiation-dominated expanding substratum can be described by the equation:

   $$\frac{d^2\Delta_B}{dt^2} + 2\left(\frac{\dot{a}}{a}\right)\frac{d\Delta_B}{dt} = 4\pi G\rho_B\Delta_B$$

   Let us now make the substitution $y = \rho_B/\rho_{\rm rad} = a/a_{\rm eq}$.

   (a) Using this substitution for y show that the equation above reduces to:

   $$\frac{d^2\Delta_B}{dy^2} + \frac{2+3y}{2y(1+y)}\frac{d\Delta_B}{dy} - \frac{3\Delta_B}{2y(1+y)} = 0$$

   (b) Show that the growing solution for this equation is $\Delta_B \propto 1 + \frac{3y}{2}$ and show that the baryonic perturbations only grew by a factor of 2.5 in amplitude throughout the entire radiation-dominated era from $y = 0$ to $y = 1$.

3. Free streaming and the damping mass. Consider a flat ΛCDM cosmology with $\Omega_{m,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$ and $h = 0.7$. Assume that the dark matter particles decouple at $z_{\rm dec} = 10^{10}$ and have a mass of $m = 2$ GeV. *Hint: Free streaming is discussed in many textbooks, for example see the Longair book Section 13.6.*

   (a) At what redshift do the dark matter particles become non-relativistic?

   (b) Show that the comoving free-streaming length at matter-radiation equality can be written as:

   $$\lambda_{\rm fs}(t_{\rm eq}) = \frac{2ct_{\rm NR}}{a_{NR}}\left[\left(\frac{a_{\rm dec}}{a_{\rm NR}}\right)^{1/2}\left\{2 + \ln\left(\frac{a_{\rm eq}}{a_{\rm dec}}\right)\right\} - 1\right],$$

   where the subscripts refer to when the particles became non-relativistic (NR), the decoupling of the particles (dec) and the matter-radiation equality (eq). *Hint: use the result that, during the radiation dominated era $a = a_{\rm NR}(t/t_{\rm NR})^{1/2}$.*

   (c) Finally, what is the ratio between $\lambda_{\rm fs}(t_{\rm eq})$ and the comoving particle horizon at $t_{\rm NR}$?

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4. Download and read the paper: *"How do galaxies populate dark matter haloes"* by Guo, White, Li & Boylan-Kolchin, 2010, MNRAS, 404, 1111 using the link on the course homepage. Based on the paper answer the questions below:

   (a) According to the paper how is the dark matter mass of the Universe distributed in different dark matter haloes as a function of their masses? How does this compare with the observed distribution of stars in galaxies? How different are these two distributions and what does this imply?

   (b) What is plotted in Fig. 1 of the paper? Why are there two different sets of symbols (black and red)? Why do the symbols not agree at lower masses and why is the error bar much smaller for the black symbols at higher masses?

   (c) What is the basic idea behind the halo occupation distribution (HOD) method used in this paper? How is the relationship between dark matter halo mass and stellar mass derived in the paper?

5. (a) Describe what Fig. 3 is showing in the paper. What are the observational data sets used for the comparison and how well does the theoretical predictions agree with the observations?

   (b) Study Table 1. According to the statistical tool developed in this paper, which are the dominant galaxies in the Local group of galaxies? Why is the 80% confidence mass interval so broad? Finally, what is meant with the timing argument and are the derived masses in the paper in agreement with this argument?

   (c) Define what is meant by galaxy formation efficiency. What does Fig 5. show? Why does the solid curve attain a maximum and decrease for both lower and higher masses? Finally, what are the different data points on the plot and why are they so far offset from the solid curve?