Galaxy formation and evolution – Problem set 2. Autumn 2020

The answers should be returned by ***Wednesday (30.9) 4pm (16.00)*** *by email to the course assistant Stuart McAlpine (stuart.mcalpine@helsinki.fi). Please put "Galaxy formation – Problem set 2" in the title of your email.*

– The problem set will be discussed on Friday (2.10) after the lecture (at 14.15) on Zoom.

1. Calculate the age of the universe at $z = 0$ and at redshifts $z = 6$ and $z = 10$ for the following models assuming a present-day Hubble parameter of $H_0 = 67.1\ \mathrm{kms^{-1}Mpc^{-1}}$:

   (a) The Einstein-de Sitter (EdS) universe, with $\Omega_{m,0} = 1$, $\Omega_{\Lambda,0} = 0$ and a flat geometry ($K = 0$), $\Omega_0 = 1$.

   (b) An open universe ($K = -1$), with $\Omega_{m,0} = 0.32$, $\Omega_{\Lambda,0} = 0$ and $\Omega_0 = 0.32$.

   (c) A flat ($K = 0$) ΛCDM model with $\Omega_{m,0} = 0.32$ and $\Omega_{\Lambda,0} = 0.68$, $\Omega_0 = \Omega_{m,0} + \Omega_{\Lambda,0} = 1$.

   In calculating the ages one can use the equations of the lecture notes, i.e. no need to rederive the equations. Interpret also the results, how different are the ages of the various models at $z = 10$ and $z = 0$. Which model gives the oldest age at $z = 0$ and why?

2. Solve the Friedmann equations by deriving an expression for the scale factor, a, as a function of time and the cosmological parameters in the following two models:

   (a) The Einstein-de Sitter (EdS) universe, with $\Omega_{m,0} = 1$, $\Omega_{\Lambda,0} = 0$ and a flat geometry ($K = 0$), $\Omega_0 = 1$.

   (b) The standard Planck ΛCDM model with $\Omega_{m,0} = 0.32$ and $\Omega_{\Lambda,0} = 0.68$, $\Omega_0 = \Omega_{m,0} + \Omega_{\Lambda,0} = 1$. *Hint: Use an appropriate subsitution in solving the integral.*

   Finally show that at early times (high redshift z, small a) the evolution of the scale factor a in the standard ΛCDM model is similar to the Einstein-de Sitter (EdS) model. Finally, how does the evolution of the scale factor differ in the two models at present and how will this difference develop in the future?

3. The discovery of the most distant confirmed object in the universe, a high-redshift galaxy named GN-z11, was announced in March 2016. This object has a confirmed redshift of $z = 11.09$ Calculate the angular-diameter and luminosity distances to this object in the following three models:

   (a) The Einstein-de Sitter (EdS) universe, with $\Omega_{m,0} = 1$, $\Omega_{\Lambda,0} = 0$ and a flat geometry ($K = 0$), $\Omega_0 = 1$.

   (b) An open universe ($K = -1$), with $\Omega_{m,0} = 0.32$ and $\Omega_{\Lambda,0} = 0$ and $\Omega_0 = 0.32$.

   (c) A flat ($K = 0$) ΛCDM model with $\Omega_{m,0} = 0.32$ and $\Omega_{\Lambda,0} = 0.68$, $\Omega_0 = \Omega_{m,0} + \Omega_{\Lambda,0} = 1$.

   Finally assume that this galaxy is a compact proto-galaxy with a very high intrinsic mean surface brightness in the rest-frame B-band of $\langle\mu\rangle_B \sim 18$ mag arcsec^{-2}. What would the observed surface brightness be, corrected for cosmological surface brightness dimming and at what wavelength would the rest-frame B-band light be observed?

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4. Download and read the paper: *"New Limits on Early Dark Energy from the South Pole Telescope"* by Reichardt, C. L, 2012, ApJL, 749, L9 using the link on the course homepage. Some parts of this paper are rather technical and require advanced knowledge of cosmological theory. In answering the questions below (4-5) the aim is to gain an understanding of the main results presented in this paper.

   (a) In this paper the authors study early dark energy models (EDE). What is meant by these EDE models and how do they differ from the standard ΛCDM model at high redshifts and at $z = 0$?

   (b) In discussing their EDE model the authors introduce two new parameters, the dark energy equation-of-state parameter at $z = 0$, w_0 and the Ω_e parameter. Define these parameters and explain what they describe.

   (c) In determining the constraints on early dark energy in the Universe the authors combine data from the Wilkinson Microwave Anisotropy Probe (WMAP) and the South Pole Telescope (SPT). Explain briefly what these instruments are and specifically what is the difference in their observed sky coverage and resolution.

5. (a) Describe what Fig 1. of the paper shows. Specifically, what is plotted on the x- and y-axes and in particular why is the SPT data crucial for detecting the EDE signal.

   (b) Explain what is meant by Silk dampening? How does the EDE influence the Silk dampening length and the observed small scale power in the cosmic microwave background?

   (c) What does Fig 3. in the paper show? Why are the coloured contours different to the black contours? What is the main conclusion of the paper, what is the constraint on the early dark energy density based on the data presented in the paper?