Galaxy formation and evolution
PAP 318, 5 op, autumn 2020
on Zoom

Lecture 5: Baryonic and dark matter models of galaxy formation – Additional notes,
02/10/2020
Lecture 5 additional notes I

- Page 3: Radiation-matter equality:

$$\rho_r = \frac{4\sigma}{c} T^4 = \frac{4\sigma}{c} T_0 (1 + z)^4$$

- Black-body radiation density:

$$\rho_r = \frac{4\sigma}{c} T^4 = \frac{4\sigma}{c} T_0 (1 + z)^4$$

- Matter density:

$$\rho_m = \Omega_0 \rho_c (1 + z)^3 c^2$$

- Page 5: The sound speed

$$c_s^2 = \frac{(\partial p/\partial T)_r}{(\partial \rho/\partial T)_r + (\partial \rho/\partial T)_m} = \frac{4}{3} \rho_r c^2}{4 \rho_r + 3 \rho_m}$$
Lecture 5 additional notes II

- Page 8: Adiabatic fluctuations in the radiation-dominated era
- Sound speed in the radiation dominated era: \( c_S = \frac{c}{\sqrt{3}} \)

- Page 8: The Jeans mass, \( \lambda_J \) the Jeans length is the diameter of the Jeans mass:

\[
M_J = \frac{4\pi}{3} \left( \frac{\lambda_J}{2} \right)^3 \rho = \frac{\pi \lambda_J^3}{6} \rho
\]

\[
\lambda_J = c_S \left( \frac{3\pi}{8G\rho} \right)^{1/2}
\]

- Page 8: Jeans mass scaling with scale factor

\[
\lambda_J \propto \rho^{-1/2}, \quad \rho \propto a^{-4}, \quad \lambda_J \propto a^2, \quad M_J \propto \lambda_J^3 \rho_b \propto a^6 a^{-3} \propto a^3
\]
Lecture 5 additional notes III

- Page 9: Adiabatic fluctuations in the matter-dominated era
- Baryonic-mass within the particle horizon, scaling with scale factor
  \[ M_{b,\text{hor}} \propto r_H^3 \rho_b, \quad r_H \propto a^{3/2}, \quad \rho_b \propto a^{-3}, \quad M_{b,\text{hor}} \propto a^{3/2} \]
- Jeans mass scaling with scale factor
  \[ M_J \propto \lambda_J^3 \rho, \quad \lambda \propto c_s \rho^{-1/2} \propto a^{-1/2} a^{3/2} \propto a, \quad M_J \propto a^3 a^{-3} \propto a^0 \]
- Page 10: Silk Dampening. Diffusion constant D:
  \[ D = \frac{1}{3} \lambda c \]
Page 11: Scaling of Silk dampening mass with scale factor

Radiation dominated era:

\[ N_e \propto (1+z)^3, \quad t \propto (1+z)^{-2}, \quad r_D \propto [(1+z)^{-2}(1+z)^{-3}]^{1/2} \propto (1+z)^{-5/2} \]

\[ M_S \propto r_D^3 \rho_b \propto (1+z)^{-15/2}(1+z)^3 \propto (1+z)^{-9/2} \]

Matter dominated era:

\[ N_e \propto (1+z)^3, \quad t \propto (1+z)^{-3/2}, \quad r_D \propto [(1+z)^{-3/2}(1+z)^{-3}]^{1/2} \propto (1+z)^{-9/4} \]

\[ M_S \propto r_D^3 \rho_b \propto (1+z)^{-27/4}(1+z)^3 \propto (1+z)^{-15/4} \]
Lecture 5 additional notes V

- Page 22-23: Instabilities in the presence of dark matter

$$\ddot{\Delta}_B + 2 \left( \frac{\dot{a}}{a} \right) \dot{\Delta}_B = 4\pi G \rho_D B a$$

- Page 23: The background is the critical $\Omega_0=1$ model.

$$a^{3/2} \frac{d}{da} \left( a^{-1/2} \frac{d\Delta}{da} \right) + 2 \frac{d\Delta}{da} = \frac{3}{2} B$$
Lecture 5 additional notes VI

- Page 23: The solution $\Delta = B(a-a_0)$ satisfies this equation:

$$a^{3/2} \frac{d}{da} \left( a^{-1/2} B \right) + 2B = \frac{3}{2} B \Rightarrow a^{3/2} \left( -\frac{1}{2} a^{-3/2} B \right) + 2B = \frac{3}{2} B$$

- Page 23: Growth of the baryon perturbations:

$$\Delta_d = B a, \quad \Delta_B = B(a - a_0), \quad \Delta_B = B a \left( 1 - \frac{a_0}{a} \right)$$

$$\frac{a_0}{a} = \frac{1 + z}{1 + z_0}, \quad \Delta_B = \Delta_D \left( 1 - \frac{z}{z_0} \right)$$