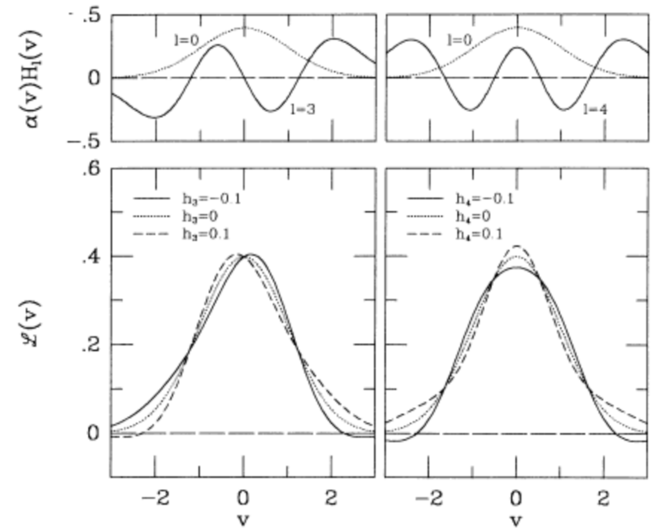
Galaxy formation and evolution
PAP 318, 5 op, autumn 2020

on Zoom

Lecture 12: Formation of elliptical galaxies – Additional notes, 27/11/2020

Lecture 12 additional notes I

- Page 5: Observed line-of-sight-velocity distributions (LOSVDs) are not perfectly Gaussian. It is customary to parameterise the LOSVD with a Gauss-Hermite series.



$$\mathcal{L}(v) = \frac{\alpha(w)}{\sigma}\left[1 + \sum_{j=3}^{N} h_j H_j(w)\right]$$

$$\alpha(w) = \frac{1}{\sqrt{2\pi}} e^{-w^2/2}, \quad w = (v - V)/\sigma$$

- Page 5: Here v is the observed LOS-velocity, V and σ are the mean and dispersion of the best fit Gaussian and H_j is the Hermite polynomial of degree j, and h_j ($j \geq 3$) is the Gauss-Hermite moment. In general h_j:s are small $|h_j| \leq 0.1$, i.e. rhe LOSVD has only relatively small departures from the Gaussian.

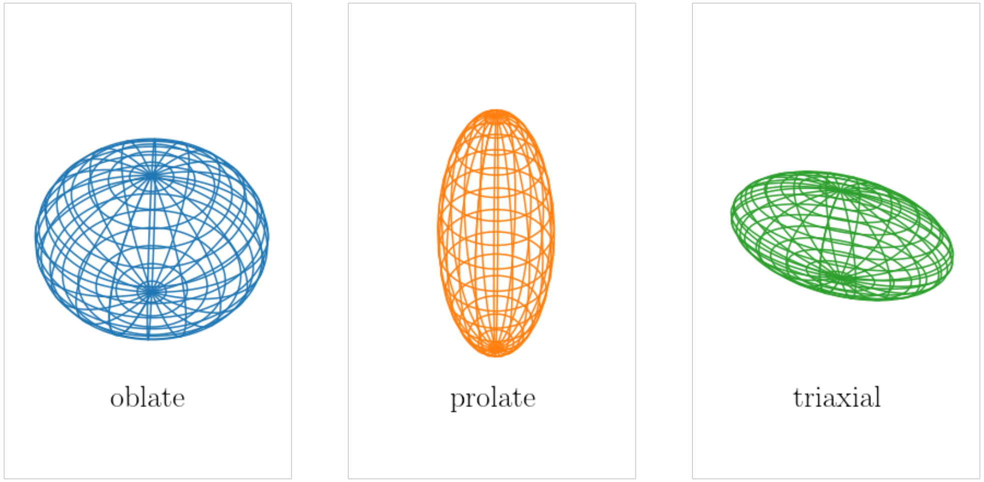
Lecture 12 additional notes II

- Page 6: Triaxial systems:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

- Page 6: Oblate systems: a=b, **c<a**:
- Page 6: Prolate systems: a=b, **c>a**:

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{c^2} = 1$$



Lecture 12 additional notes III

- Page 9: The Jeans equation can be used to constrain the mass distribution from the observed kinematics (see the Galactic dynamics course, Spring 2021 for further details):

$$\frac{1}{\rho}\frac{d(\rho\langle v_r^2\rangle)}{dr} + 2\beta\frac{\langle v_r^2\rangle}{r} = -\frac{d\Phi}{dr}, \qquad \beta(r) = 1 - \langle v_\vartheta^2\rangle/\langle v_r^2\rangle$$

- Page 9: The β-parameter can also be expressed as:

$$\beta = 1 - \frac{\sigma_{\rm tan}^2}{\sigma_{\rm rad}^2} = 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{2\sigma_r^2}, \qquad \beta = [-\infty, 1]$$

- Page 9: For an isotropic system ($\sigma_\theta = \sigma_\phi = \sigma_r$) $\beta=0$. For radially biased systems $\beta>0$ and for tangentially biased systems $\beta<0$.

Lecture 12 additional notes IV

- Page 12: The tensor virial theorem can be expressed as:

$$2T_{ij} + \Pi_{ij} + W_{ij} = 0$$

$$T_{ij} = \frac{1}{2}\int \rho \langle v_i \rangle \langle v_j \rangle d^3\vec{x}$$

$$\Pi_{ij} = \int \rho \sigma_{ij}^2 d^3\vec{x}, \quad \sigma_{ij}^2 = \langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle$$

$$W_{ij} = -\int \rho x_i \frac{\partial \Phi}{\partial x_j} d^3\vec{x}$$

- Page 12: The δ-parameter is a measure of the anisotropy in the velocity dispersion in an axisymmetric system that rotates about its symmetry axis (z-axis in this case): $\delta = 1 - \Pi_{zz}/\Pi_{xx} < 1$

Lecture 12 additional notes V

- Page 13: Violent relaxation: In a star moves in fixed potential Φ its energy (ε) is constant but if Φ is a function of both space and time, $\Phi(x,t)$ the energy is not constant:

$$\epsilon = \frac{1}{2}v^2 + \Phi, \quad \frac{d\epsilon}{dt} = \frac{\partial \epsilon}{\partial v}\frac{d\vec{v}}{dt} + \frac{\partial \epsilon}{\partial \Phi}\frac{d\Phi}{dt}$$

$$\frac{d\epsilon}{dt} = -\vec{v}\cdot\nabla\Phi + \frac{d\Phi}{dt} = -\vec{v}\cdot\nabla\Phi + \frac{\partial\Phi}{\partial t} + \vec{v}\cdot\nabla\Phi = \frac{\partial\Phi}{\partial t}$$

- Page 22: Sizes of ellipticals: Energy in virial equilibrium:

$$E = T + W, \quad 2T + W = 0, \quad T = -\frac{W}{2}, \Rightarrow E = \frac{W}{2}$$

Lecture 12 additional notes VI

- Page 22-23: The sizes of ellipticals:

$$E_f = \eta E_i, \Rightarrow \xi_f \frac{GM_{\rm gas}^2}{2r_e} = \eta \frac{3}{5} \frac{GM_{\rm gas}^2}{f_{\rm gas} r_t}$$

$$\eta = \frac{5}{3} \cdot \xi_f f_{\rm gas} \frac{r_t}{2r_e} = \frac{5}{3} \cdot 0.6 \cdot \frac{2r_{\rm vir}}{2r_e} f_{\rm gas} = f_{\rm gas} \frac{r_{\rm vir}}{r_e}$$

- Page 23: The final equation of η is derived, assuming that the average density of a dark matter halo is100 times the critical density for closure and the observed size-mass relationship from the Sloan Digital sky survey:

$$\eta = 6.9 \left(\frac{f_{\rm gas}}{0.15} \right)^{0.44} \left(\frac{M_{\rm vir}}{10^{12} h^{-1} M_\odot} \right)^{-0.23}$$