Galaxy formation and evolution
PAP 318, 5 op, autumn 2020

on Zoom

Lecture 11: Galaxy interactions and transformations – Additional notes, 20/11/2020

Lecture 11 additional notes I

- Page 4: Interaction timescale:

$$t_{\rm enc} \sim \frac{R_{\rm max}}{V}, \quad t_{\rm tide} \sim \frac{R_{\rm gal}}{\sigma}, \quad t_{\rm enc} \gg t_{\rm tide} \Rightarrow \quad \frac{R_{\rm max}}{V} \gg \frac{R_{\rm gal}}{\sigma}$$

- Page 7: High-speed encounters: The angle ϕ is the angle between the vectors **r** and **R** in the Figure on page 3 and we can use the cosine rule for a triangle:

$$\Phi_P = -\frac{GM_P}{|\vec{r} - \vec{R}|}, \quad |\vec{r} - \vec{R}| = \sqrt{R^2 - 2rR\cos\phi + r^2}$$

Lecture 11 additional notes II

- Page 8: High-speed encounters: r<<R -> r^2~0.
- Page 8: The denominator can be expresses as below and expanded with a Taylor series:

$$|\vec{r}-\vec{R}| = \sqrt{R^2 - 2rR\cos\phi} = R\left[1+\left(-\frac{2r}{R}\cos\phi\right)\right]^{1/2}, \quad x = \left(-\frac{2r}{R}\cos\phi\right)$$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{15}{48}x^3 + \ldots$$

- Page 9: Impulse approximation:
  In spherical symmetry all axis contribute equally to the radius:

$$< x^2 >=< y^2 >=< z^2 >= \frac{1}{3} < r^2 >, \Rightarrow < x^2 + z^2 >=< x^2 > + < z^2 >= \frac{2}{3} < r^2 >$$

Lecture 11 additional notes III

- Page 11: Impulsive heating and virial equilibrium:

$$E_{\rm tot} = E_{\rm p} + E_{\rm k}, \quad 2E_{\rm k} + E_{\rm p} = 0, \quad E_{\rm p} = -2E_{\rm k}, \quad \Rightarrow E_{\rm tot} = -E_{\rm k}$$

- Page 12: Tidal stripping: Taylor expansion of term

$$\frac{1}{(1+(-x))^2} \approx 1 - 2x + ..., \quad x = (r/R)$$

$$\frac{GM}{(R-r)^2} = \frac{GM}{R^2[1-(r/R)]^2} \approx \frac{GM}{R^2}\left(1 - 2\frac{r}{R}\right)$$

Lecture 11 additional notes IV

- Page 17: Dynamical friction: full formula

$$\vec{F}_{\rm df} = M_S \frac{d\vec{v}_S}{dt} = -16\pi^2 G^2 M_S^2 m \log\Lambda \left[\int_0^{v_S} f(v_m) v_m^2 dv_m\right] \frac{\vec{v}_S}{v_S^3}$$

where $f(v_m)$ is the phase-space density.

$$\rho(< v_S) = m \int_0^{v_S} 4\pi f(v_m) v_m^2 dv_m$$

- Page 17: For small velocities $f(v_m) \sim f(0)$

$$\vec{F}_{\rm df} = M_S \frac{d\vec{v}_S}{dt} \propto f(0) \left[\int_0^{v_S} v_m^2 dv_m\right] \frac{\vec{v}_S}{v_S^3} \propto v_S$$

Lecture 11 additional notes V

- Page 17: For large velocities (n is the number density)

$$\int_0^{\infty} f(v_m) v_m^2 dv_m = \frac{n}{4\pi}$$

- Page 17: For large velocities the dynamical friction

$$\vec{F}_{\rm df} = M_S \frac{d\vec{v}_S}{dt} \propto \frac{n}{4\pi} \frac{\vec{v}_S}{v_S^3} \propto v_S^{-2}$$

- Page 19: Orbital decay:

$$F_{df} = M_S \frac{d\vec{v}_S}{dt}, \quad \frac{d\vec{v}_S}{dt} = \frac{F_{df}}{M_S}$$

Lecture 11 additional notes VI

- Page 20: Orbital decay: The subject mass continues to orbit with the constant velocity v_S while it spirals in:

$$\int r dr = \int -0.428 \frac{GM_S}{V_c} \ln \Lambda dt$$

$$\frac{1}{2} r^2 = -0.428 \frac{GM_S}{V_c} \ln \Lambda \; t_{\mathrm{df}}$$

$$t_{\mathrm{df}} = \frac{1.17}{\ln \Lambda} \frac{r_i^2 V_c}{GM_S} \rightarrow t_{\mathrm{df}} = \frac{1.17}{\ln \Lambda} \left(\frac{r_i}{r_h} \right)^2 \left(\frac{M_h}{M_S} \right) \frac{r_h}{V_c}$$

$$V_c^2 = \frac{GM_h}{r_h}, \quad G = \frac{V_c^2 r_h}{M_h}$$

Lecture 11 additional notes VII

- Page 21: Singular isothermal spheres:

$$\rho(r) = \frac{V_h^2}{4\pi G r^2}, \quad r_h = \sqrt{\frac{200}{\Delta_h \Omega_m}} \frac{V_h}{10H(z)}, \quad V_h = \sqrt{\frac{GM_h}{r_h}}$$