Galaxy formation and evolution
PAP 318, 5 op, autumn 2020

on Zoom

Lecture 10: Formation of disk galaxies – Additional notes, 13/11/2020

Lecture 10 additional notes I

- Page 4-5: Flat rotation curves imply the existence of dark matter, as the surface mass density of the luminous part is exponentially declining, as seen in the surface brightness profile of disc galaxies.

$$v(r) = \sqrt{\frac{GM(r)}{r}}, \quad v(r) \sim \text{ const}, \Rightarrow \mathrm{M(r)} \propto \mathrm{r}$$

- Page 6: Bessel functions are canonical solutions y(x) of Bessel's differential equation, α gives the order of the Bessel function:

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - \alpha^2) y = 0$$

Lecture 10 additional notes II

- Page 6: Bessel functions of the first kind $J_\alpha(x)$:

$$J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(m+\alpha+1)} \left(\frac{x}{2}\right)^{2m+\alpha}$$

- Page 6: Modified Bessel functions are solutions to:

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} - (x^2 - \alpha^2)y = 0$$

$$I_\alpha = i^{-\alpha} J_\alpha(ix), \quad K_\alpha(x) = \frac{\pi}{2} \frac{I_{-\alpha}(x) - I_\alpha(x)}{\sin(\alpha x)}$$

Lecture 10 additional notes III

- Page 11: Angular momentum transport:

$$J = m\vec{v} \times \vec{r}, \quad v = \sqrt{GM/R}, \quad J = \delta M (GMR)^{1/2}$$

- Page 13: Disk mass derivation:

$$M_{\rm vir} = \frac{4\pi}{3}\rho_{\rm vir} r_{\rm vir}^3, \quad \rho_{\rm vir} = \frac{3H(z)^2}{8\pi G} \times \Delta_{\rm vir}, \quad v_{\rm vir}^2 = \frac{GM_{\rm vir}}{r_{\rm vir}}$$

$$\Delta_{\rm vir} = 200, \quad M_{\rm vir} = \frac{4\pi}{3}\frac{3}{8\pi G}H^2(z) \times 200 \frac{G^3 M_{\rm vir}^3}{v_{\rm vir}^6}$$

$$\Rightarrow M_{\rm vir} = \frac{v_{\rm vir}^3}{10GH(z)}$$

Lecture 10 additional notes IV

- Page 14: Total energy using the virial theorem:

$$E_{\rm tot} = U + K, \quad 2U + K = 0, \quad U = -2K, \ \Rightarrow E_{\rm tot} = K - 2K = -K$$

- Page 15: Disk angular momentum:

$$J_d = 2M_d R_d V_{\rm vir}, \quad J_d = j_d J_{\rm vir}$$

$$J_{\rm vir} = \lambda G M_{\rm vir}^{5/2} |E|^{-1/2} = \lambda G M_{\rm vir}^{5/2} \sqrt{2} M_{\rm vir}^{-1/2} V_{\rm vir}^{-1}$$

$$J_d = \sqrt{2} j_d \lambda M_{\rm vir}^2 G V_{\rm vir}^{-1}, \quad G = \frac{v_{\rm vir}^2 R_{\rm vir}}{M_{\rm vir}}$$

Lecture 10 additional notes V

- Page 15: Disc scale radius:

$$J_d = \sqrt{2} j_d \lambda M_{\rm vir} R_{\rm vir} V_{\rm vir} \quad \& \quad J_d = 2 M_d R_d V_{\rm vir}$$

$$2 M_d R_d V_{\rm vir} = \sqrt{2} j_d \lambda \frac{M_d}{m_d} R_{\rm vir} V_{\rm vir}$$

$$R_d = \frac{1}{\sqrt{2}} \lambda \left(\frac{j_d}{m_d} \right) R_{\rm vir}$$

- Page 16: Total rotation curve is a combination of the disc component, $V_{c,d}$ and the adiabatically contracted dark matter halo component, $V_{c,h}$.

$$V_c^2(R) = V_{c,d}^2(R) + V_{c,h}^2(R) = V_{c,d}^2(R) + \frac{G M_{h,ac}(R)}{R}$$

Lecture 10 additional notes VI

- Page 17: Adiabatic contraction. Assuming that rM(r) is conserved and that initially the baryons and dark matter follow the same NFW density profile we get after disc formation:

$$M_f(r_f) = M_d(r_f) + [1 - m_d]M_i$$