1. Let $M$ be complete with $K(σ) \leq 0$ for every 2-planes $σ \subset T_pM$, $\forall p \in M$. Prove that $\forall p \in M$, $\exp_p : T_pM \to M$ is a local diffeomorphism.

2. Let $e_1, \ldots, e_n$ be an orthonormal basis of $T_pM$, $(U, ϕ)$ the corresponding normal chart at $p$, and $g_{ij}$ the corresponding component functions of the Riemannian metric. Prove that
   
   $g_{ij}(\exp_p v) = δ_{ij} - \frac{1}{3} \langle R(e_i, v)e_j, e_j \rangle + O(|v|^3), \quad \text{for } \exp_p v \in U. \quad \text{[Recall Exercise 9/5.]}$

3. Let $A_{ij} : \mathbb{R} \to \mathbb{R}$, $i, j = 1, \ldots, n$, be smooth mappings and denote $A = (A_{ij})$. Suppose that $\det A(0) > 0$. Prove that the function $\det A$ has the expansion
   
   \[
   \frac{\det A(t)}{\det A(0)} = 1 + t \cdot \text{tr}(A'A^{-1})(0) + \frac{t^2}{2} \left( \text{tr}(A''A^{-1})(0) - \text{tr}((A'A^{-1})^2)(0) + (\text{tr}(A'A^{-1})(0))^2 \right) + O(t^3)
   \]
   
   in a neighborhood of 0. \quad \text{[Recall Exercise 6/5.]}$

4. Prove that in the situation of Exercise 2,
   
   $\det(g_{ij}(\exp_p v)) = 1 - \frac{1}{3} \text{Ric}(v, v) + O(|v|^3)$
   
   for $\exp_p v \in U$.

5. Let $γ : I \to M$ be a geodesic, $0 \in I$, and $p = γ(0)$. Prove that, for every $h \in C^\infty(p)$, we have
   
   $(h \circ γ)'(0) = \text{Hess} h(\dot{γ}_0, \dot{γ}_0).$